

Tidal resonance in icy satellites with subsurface oceans

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Abstract

We extend the formulation for tidal deformation based on viscoelasto-gravitational theory to incorporate inertial effects. Although ocean dynamics is treated in a simplified fashion, we find a resonant configuration when the phase velocity of ocean gravity waves is similar to the orbital velocity. A static ocean formulation gives an accurate result only if the ocean thickness is larger than the resonant thickness by a factor of about one hundred, of order 10 km. The resonant configuration strongly depends on the properties of the shell, demonstrating the importance of the presence of a shell on tidal dissipation.

1. Introduction

Tidal dissipation is one of the major heat sources for the evolution of planetary bodies, particularly the satellites of the giant planets. Based on internal thermal and structural modeling, icy satellites of Jupiter and of Saturn are expected to possess an internal ocean underneath an icy shell. Pluto may also possess a subsurface ocean, and tidal dissipation due to the orbital motion of its satellite, Charon, may have heated Pluto in the past. Thus, a detailed investigation of the evolution of planetary bodies in the outer Solar System should consider tidal dissipation using an interior model consisting of an outer solid layer, internal liquid layer(s), and a solid (or liquid) core.

Previous studies of tidal dissipation in icy satellites using spherically-symmetric models can be classified into two types: those considering tidal dissipation in the solid part, and those considering tidal dissipation in the ocean. The former type has been applied to many satellites and planets [e.g., 1], though the effect of an internal liquid layer on tidal deformation has usually been treated in a simplified fashion. On the other hand, the latter type considers ocean dynamics [e.g., 2]. One important assumption in the latter models is that the surface topography follows an equipotential surface. This requires either that the ocean is at the surface or that the ice shell overlying the ocean is soft. In reality, none of the icy satellites has a surface ocean, and in some cases (such as Titan) the ice shell may not be sufficiently soft because the viscosity of ice is very high at low temperatures.

In this study, we extend the viscoelastogravitational theory and obtain a comprehensive equation system that can account for a thin subsurface ocean in viscoelastic planets (and satellites). We then apply our theory to icy satellites and investigate the effect of an icy shell on the resonant configuration.

2. Theory

Our formulation is based on the well-established elasto-gravitational theory considering deformation of a spherically symmetric, non-rotating, elastic, and isotropic body. In this theory, three equations (i.e., the equation of momentum conservation, the Poisson equation for the gravitational field, and the constitutive equation) are solved applying the spherical harmonic expansion. If we include and exclude the inertial term in the equation of momentum conservation, we obtain four- and two-component equation systems for a liquid layer, respectively. The latter equation system has been widely used in previous studies. In the following, we call the formulation using the four-component equation system a dynamic formulation while the twocomponent system a static formulation.

3. Tidal resonance

We first investigate the inertial effects using a fivelayer Ganymede model [3]. Figure 1 shows the real part of the tidal Love number h_2 as a function of ocean thickness, H_{ocean} . This figure clearly shows that our dynamic formulation leads to a resonance when $H_{\text{ocean}} \sim 0.1$ km. We found that this thickness is close to the thickness that results in a phase velocity of gravity waves ($\approx \sqrt{gH_{\text{ocean}}}$, g is gravitational acceleration) similar to the orbital velocity (= ω_{orb}/k , k is wavenumber). For this Ganymede



Figure 1: Real part of h_2 as a function of ocean thickness for a Ganymede model. The vertical dashed lines indicate the resonant thickness $H_{\rm res} \approx 85.2$ m.

model, a resonant thickness $H_{\rm res} \approx 85$ m. Figure 1 illustrates that an increase in the real part of h_2 can be seen even if $H_{\rm ocean}$ is several km. Thus, a static formulation would give a sufficiently accurate result only if $H_{\rm ocean} \gtrsim 100 H_{\rm res}$; a dynamical formulation should be used if $H_{\rm ocean} \lesssim 100 H_{\rm res}$.

4. Effect of a lid on tidal resonance

We then investigate the effect of varying lid parameters on the resonant configuration. Here, we use a very simple, incompressible three-layer Enceladus model, consisting of a viscoelastic shell, a liquid ocean, and an elastic mantle.

Figure 2 demonstrates the effect of a lid on the tidal resonance. If the shell thickness $(D_{\rm shell})$ is small or the shell viscosity $(\eta_{\rm ice})$ is moderate or high, there is always one resonant configuration (Figure 2 (c) and (d)). For a given $\eta_{\rm ice}$, an increase in $D_{\rm shell}$ leads to a smaller resonant ocean thickness. Because of nonzero rigidity, an icy shell acts as a membrane resisting deformation. This is similar in principle to surface tension acting at a liquid surface. The phase velocity of gravity waves taking surface tension into account is $\approx \sqrt{gH_{\rm ocean} (1 + Tk^2/\rho g)}$, where T is surface tension. Thus, an increase in T requires a decrease in $H_{\rm ocean}$ to lead to the same velocity.



Figure 2: Absolute value of k_2 as a function of ocean thickness for an Enceladus model. The dotted curves are results for a surface ocean case ($D_{\rm shell} = 0$ km). The vertical dashed lines indicate the resonant thickness $H_{\rm res} \approx 263$ m.

5. Conclusions

We found that a dynamic ocean leads to a resonance while a static ocean does not. This resonance would be important for a satellite with a thin subsurface ocean since it would lead to significantly enhanced tidal heating in the solid lid. The static ocean formulation, which has been used in previous studies, would give an accurate Love number only if the ocean thickness is larger than the resonant thickness by a factor of about one hundred (or ~10 km). A thicker or more rigid shell leads to a thinner resonant ocean thickness. These results highlight the importance of the effects of a solid lid for tidal dissipation in icy satellites with a subsurface ocean.

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References

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