# The fossil Oort Cloud and the dynamics beyond Neptune

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# Summary

# Introduction

# The fossil Oort Cloud

**Fossil Oort Cloud :** intermediate region between the inner Solar System and the Oort Cloud (stability on a Gyr time scale).

- $a \in [100, 1000]$ AU
- perihelion $q$ well detached from the planets

*Problem : are "classical planetary dynamics" enough to produce a fossil Oort Cloud from the planetary region ?*

**Kozai mechanism beyond Neptune :**

- Thomas and Morbidelli 1996
- Gallardo 2006
- Gomes et al. 2005
- Gallardo et al. 2012

# How far can we go ?

*Goal :* ***quantify*** *what perihelion distances we can possibly reach,* ***and how****.*

**Method :** secular theory for a particle perturbed by some planets.

- integrable approximation
- idealized planetary motion
- comparison with full numerical integration (*do we capture the essence of the real problem ?*)

**Planetary orbits used :**

- the four giant planets on circular, coplanar orbits

# No mean motion resonance

# Hamiltonian for the problem

**Hamiltonian function in heliocentric coordinates :** $\mu = \mathcal{G}M_\odot$

$$\mathcal{H} = -\frac{\mu^2}{2L^2} + \sum_{i=1}^{N} n_i\,\Lambda_i - \sum_{i=1}^{N} \mathcal{G}m_i \left( \frac{1}{|\mathbf{r} - \mathbf{r}_i|} - \mathbf{r} \cdot \frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} \right)$$

**Canonical coordinates :**

$$\begin{cases} l = M \\ g = \omega \\ h = \Omega \\ \lambda_1, \lambda_2 \ldots \lambda_N \end{cases} \quad \text{and} \quad \begin{cases} L = \sqrt{\mu a} \\ G = \sqrt{\mu a\,(1 - e^2)} \\ H = \sqrt{\mu a\,(1 - e^2)}\cos I \\ \Lambda_1, \Lambda_2 \ldots \Lambda_N \end{cases}$$

**Secular Hamiltonian (1st order)** : average of $\mathcal{H}$ with respect to the independent fast angles $l$ and $\lambda_1, \lambda_2 \ldots \lambda_N$.

# Secular non-resonant Hamiltonian

**General form in Delaunay coordinates :** $\mathcal{H}_s = \mathcal{H}_s(L, G, H, g)$

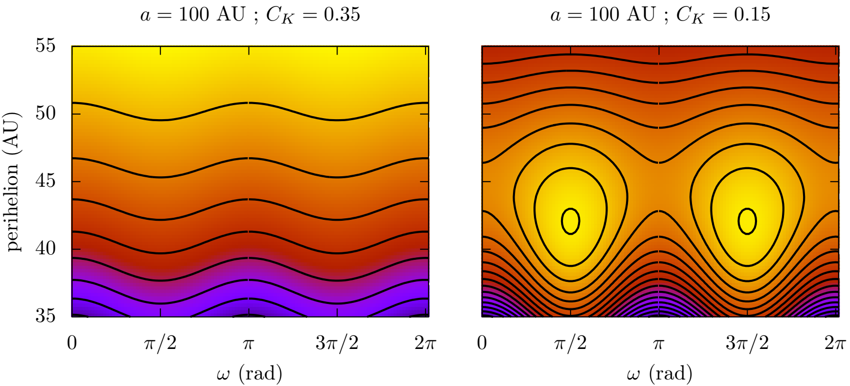
$\Longrightarrow$ secular momenta conserved : $\begin{cases} L = \sqrt{\mu\, a} \\ H = \sqrt{\mu\, a\,(1-e^2)}\,\cos I \end{cases}$

- The secular $a$ is conserved.
- The secular $e$ and $I$ are linked by the *Kozai constant* :

$$C_K = (1-e^2)\cos^2 I$$

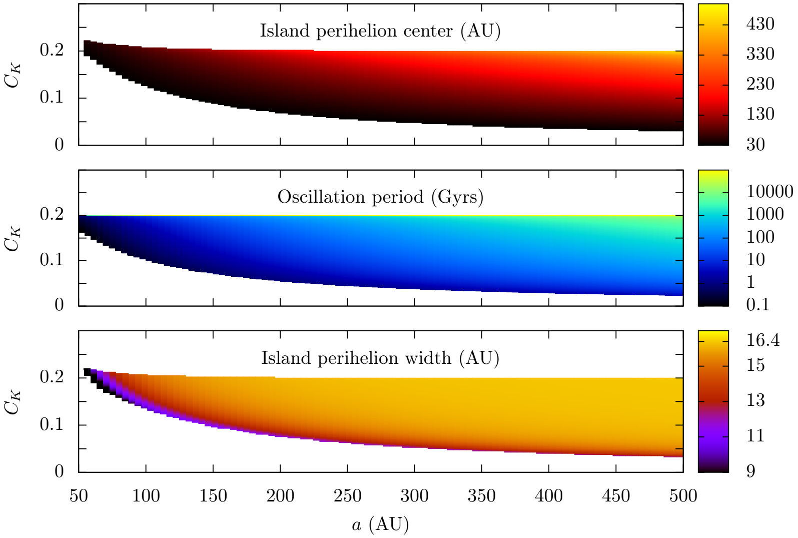
**One degree of freedom :** with $a$ and $C_K$ as parameters, the Hamiltonian is described by its level curves in the $(q, \omega)$ plan.

# Two kinds of motion




$C_K > 1/5$ $C_K < 1/5$

# Range of possible behaviours



# Single mean motion resonance

# Change of coordinates

**Principal resonant angle :** $\sigma = k\,\lambda - k_p\,\lambda_p - (k - k_p)\,\varpi$
with $k, k_p \in \mathbb{N}$ and $k > k_p$.

- **New set of angle coordinates :**

$$\mathbf{A}\begin{pmatrix} M \\ \lambda_p \\ \omega \\ \Omega \\ \{\lambda_{i\neq p}\} \end{pmatrix} = \begin{pmatrix} \sigma \\ \gamma \\ u \\ v \\ \{\lambda_{i\neq p}\} \end{pmatrix} \begin{array}{l} \longleftarrow \text{ resonant angle} \\ \longleftarrow \text{ fast angle} \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \longleftarrow \text{ no change} \end{array}$$

- **Conjugated momenta :** $(\Sigma, \Gamma, U, V, \{\Lambda_{i\neq p}\})$ using $(\mathbf{A}^T)^{-1}$

**Semi-secular Hamiltonian (1st order)** : average of $\mathcal{H}$ with respect to the independent fast angles $\gamma$ and $\{\lambda_{i\neq p}\}$.

# Semi-secular resonant Hamiltonian

**General form** (dropping constant parts) : $\mathcal{H}_{ss} = \mathcal{H}_{ss}(\Sigma, U, V, \sigma, u)$

$$\Longrightarrow \text{ quantity conserved : } V = \sqrt{\mu\, a}\left(\sqrt{1-e^2}\cos I - k_p/k\right)$$

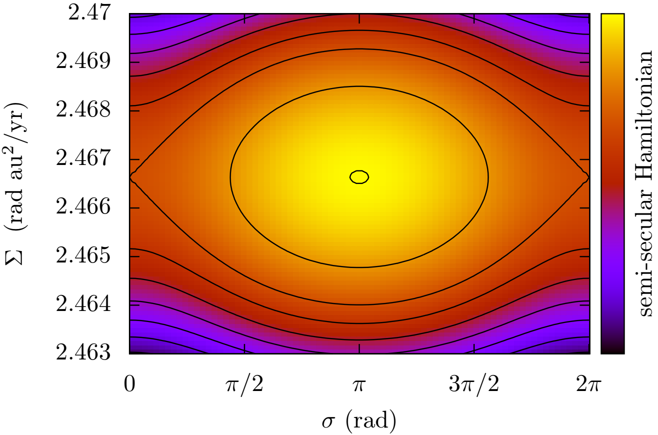
*A second averaging step would be required to get a one-degree-of-freedom secular Hamiltonian.*

**Adiabatic invariant theory :**

In that region of the Solar System, the $\sigma$ oscillation time-scale is always much smaller than $u$ secular period.

# Two separated time-scales

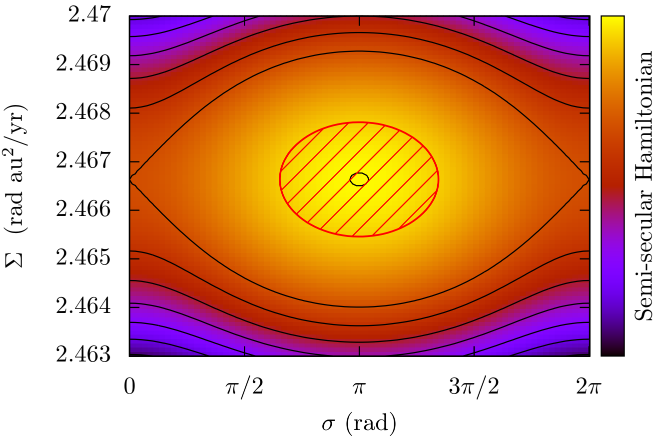
*As $T_u \gg T_\sigma$, the semi-secular evolution is well described in the $(\Sigma, \sigma)$ plan, for a fixed value of $(U, u)$.*



# The adiabatic invariant

**Action-angle coordinates of $\mathcal{H}_{ss}$ for a fixed value of $(U, u)$ :**

- $\theta$ = mean motion on the trajectory
- $J \propto$ enclosed area = *adiabatic invariant*



# Secular resonant Hamiltonian

**Semi-secular Hamiltonian in the new coordinates :**

$$\mathcal{H}_{ss}(J,U,\theta,u) = \mathcal{K}_0(J) + \epsilon\,\mathcal{K}_1(J,U,\theta,u)$$

where $\epsilon$ is related to the frequency ratio secular/semi-secular.

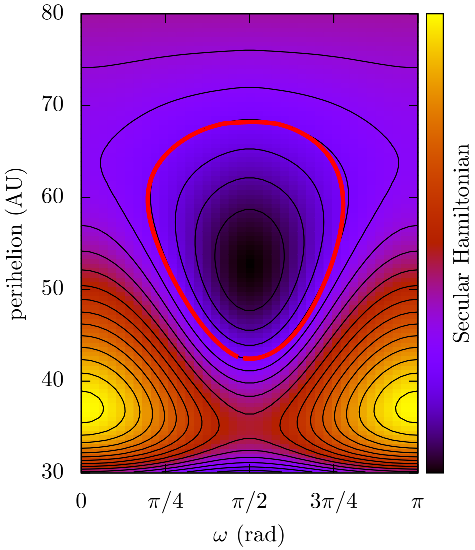
**Secular Hamiltonian (1st order in $\epsilon$) :** after averaging over $\theta$, the action $J$ is secularly conserved. We finally get a 1-DoF system :

$$\mathcal{H}_s = \mathcal{H}_s(J,U,u)$$

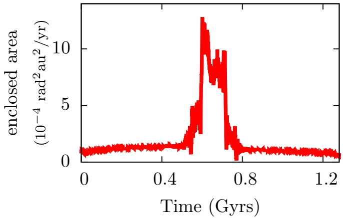
**Validity of the adiabatic approximation :**

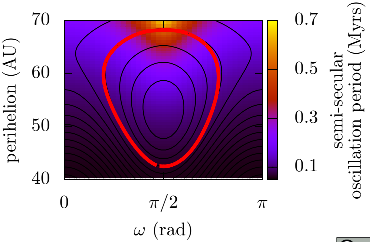
- no separatrix approaching ($\epsilon$ must remain small)
- no separatrix crossing (possible chaotic jumps in $J$)

# Example : Neptune 2:37 resonance




## Constancy of $J$ ?



# Conclusion

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**Secular dynamics beyond Neptune :**

- small perihelion excursions without resonance ($\Delta q < 16.4$ AU)
- very important variations possible in the resonant case (quantification and classification in progress...)

*In the resonant case, a special care must be taken about the relevance of the adiabatic approximation used.*

**Next steps :**

1) atlas of mean motion resonances and associated $q$ excursions
2) study of the chaos induced by separatrix crossings
3) possibility of applications to known objects (Sedna, $2012\text{VP}_{113}$)