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Time-dependent solution for reorientation of rotating tidally deformed visco-elastic bodies

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Abstract

A new semi-analytical method is established to calculate the time-dependent solution for reorientation of co-rotating tidally deformed bodies. Compared with the widely used fluid limit solution by [*Matsuyama and Nimmo* (2007)], a more accurate time-dependent reorientation path can be obtained. We will use this method to constrain the interior viscosities of Pluto based on recent discoveries from the New Horizons mission.

1. Introduction

Many icy satellites or planets contain features which suggest a (past) reorientation of the body, such as the tiger stripes on Enceladus and the heart-shaped Sputnik Planum on Pluto. Most of these icy bodies are tidally locked and this creates a large tidal bulge which is about three times as large as its centrifugal (equatorial) bulge. To study the reorientation of such rotating tidally deformed body is complicated and most previous studies apply the so-called fluid limit method. The fluid limit approach ignores the viscous response of the body and assumes that it immediately reaches its fluid limit when simulating the reorientation due to a changing load. As a result, this method can only simulate cases when the change in the load is much slower than the dominant viscous modes of the body. For other kinds of load, for instance a Heaviside load due to an impact which creates an instant relocation of mass, it does not give us a prediction of how the reorientation is accomplished (e.g. How fast? Along which path?). Here, we seek a dynamic solution which can provide this answer.

2. Method

2.1 Moment of Inertia

The change in the inertia tensor for a centrifugally and tidally deformed body is given by

$$\begin{aligned}\Delta I_{ij}(t) = &\frac{k^T(t)a^5}{3G} * [\omega_i(t)\omega_j(t) - \frac{1}{3}\omega^2(t)] \\ &+\frac{k^T(t)a^5}{3G} * [\bar{\omega}_i(t)\bar{\omega}_j(t) - \frac{1}{3}\bar{\omega}^2(t)] \\ &+\delta(t) + k^L(t)] * C_{ij}(t)\end{aligned} \quad (1)$$

where $\boldsymbol{\omega}$ and $\bar{\boldsymbol{\omega}}$ are vectors of rotation and tidal potential, the magnitude of $\bar{\boldsymbol{\omega}}$ is usually $\sqrt{3}$ times $\boldsymbol{\omega}$ for most icy moons. This equation can be solved analytically by linear change assumptions or calculated numerically with a finite-element package.

2.2 Liouville equation

With the information of the deformation, the reorientation is obtained by combining the perturbation of rotational and tidal axes. Both can be obtained from a general linearized Liouville equation

$$m_1(t) = \frac{\Delta I_{13}(t)}{C - A} + \frac{C\Delta\ \dot{I}_{23}\ (t)}{\Omega(C - A)(C - B)} \quad (2a)$$

$$m_2(t) = \frac{\Delta I_{23}(t)}{C - B} - \frac{C\Delta\ \dot{I}_{13}\ (t)}{\Omega(C - A)(C - B)} \quad (2b)$$

Here m_1 and m_2 are the perturbation in the plane perpendicular to the rotational and tidal axes.

2.3 An iterative procedure

An iterative algorithm which is shown in [*Hu al.* (2017)] is applied in each time step:

Algorithm

1. Assume that the step i, from time t_i to t_{i+1}, starts with the direction of the rotational axis given by $\boldsymbol{\omega_r^i} = \Omega_r^i(\omega_1^i, \omega_2^i, \omega_3^i)^T$ and the direction of the tidal axis by $\boldsymbol{\omega_t^i} = \Omega_t^i(\omega_4^i, \omega_5^i, \omega_6^i)^T$ in which $(\omega_1^i, \omega_2^i, \omega_3^i)^T$ and $(\omega_4^i, \omega_5^i, \omega_6^i)^T$ are unit column

vectors which satisfy $\omega_1^i\omega_4^i + \omega_2^i\omega_5^i + \omega_3^i\omega_6^i = 0$. For the first iteration, we assume that the rotation and tidal axes in this step do not change: $\boldsymbol{\omega_r^{i+1}} = \boldsymbol{\omega_r^i}$ and $\boldsymbol{\omega_t^{i+1}} = \boldsymbol{\omega_t^i}$.

2. Apply both the centrifugal and tidal potential to the model and solve the equation 1. Obtain the total change in the inertia tensor and its derivative as $\Delta\mathbf{I}$ and $\Delta\dot{\mathbf{I}}$. The coordinate transformation matrix from the body-fixed to the bulge-fixed coordinate system is given by

$$\mathbf{U} = [\boldsymbol{\omega_t^{i+1}}, \boldsymbol{\omega_r^{i+1}} \times \boldsymbol{\omega_t^{i+1}}, \boldsymbol{\omega_r^{i+1}}] \tag{3}$$

The local values of the inertia tensor for the centrifugal part are obtained by $\Delta\mathbf{I}_1 = \mathbf{U}^T\Delta\mathbf{I}\,\mathbf{U}$ and $\Delta\dot{\mathbf{I}}_1 = \mathbf{U}^T\Delta\dot{\mathbf{I}}\,\mathbf{U}$. The corresponding inertia tensors for calculating tidal perturbation are $\Delta\mathbf{I}_2 = -\mathbf{S}^T\Delta\mathbf{I}_1\,\mathbf{S}$ and $\Delta\dot{\mathbf{I}}_2 = -\mathbf{S}^T\Delta\dot{\mathbf{I}}_1\,\mathbf{S}$. $\mathbf{S}$ is the transformation matrix between frames where the Z-axis is the rotational axis and the tidal axis.

3. Apply equation 2 to $\Delta\mathbf{I}_1$ and $\Delta\dot{\mathbf{I}}_1$ and obtain the perturbation for the rotational axis as $\Omega_1(m_1, m_2, m_3)$. Apply equation 2 to $\Delta\mathbf{I}_2$ and $\Delta\dot{\mathbf{I}}_2$ and obtain the perturbation for the tidal axis as $\Omega_2(m_1', m_2', m_3')$. Then we have the perturbed Z- and X-axis as $\mathbf{Z}' = \Omega_1(m_1, m_2, 1 + m_3)^T$ and $\mathbf{X}' = \Omega_2(1 + m_3', m_2', -m_1')^T$. We normalize these vectors as $\mathbf{Z}' = \Omega_r^{i+1}\bar{\mathbf{Z}}'$ and $\mathbf{X}' = \Omega_t^{i+1}\bar{\mathbf{X}}'$. The local coordinate transformation matrix from the bulge-fixed frame at time t_i to the new frame at time t_{i+1} is obtained as $\mathbf{V} = [\bar{\mathbf{X}}', \bar{\mathbf{Z}}' \times \bar{\mathbf{X}}', \bar{\mathbf{X}}' \times (\bar{\mathbf{Z}}' \times \bar{\mathbf{X}}')]$. The updated direction of the rotational and tidal axes in the original body-fixed coordinates are obtained as

$$\boldsymbol{\omega_r^{i+1}} = \Omega_r^{i+1}\mathbf{U}\mathbf{V}[0, 0, 1]^T \tag{4a}$$

$$\boldsymbol{\omega_t^{i+1}} = \Omega_t^{i+1}\mathbf{U}\mathbf{V}[1, 0, 0]^T \tag{4b}$$

4. Substitute $\boldsymbol{\omega_r^{i+1}}$ and $\boldsymbol{\omega_t^{i+1}}$ in step 2 until the results converge.

3. Results

We use a Triton model which contains a 10 km lithosphere for demonstration. For a Heaviside load, the fluid limit solution can only provide the end position of a reorientation. We want to demonstrate the reorientation due to accumulated ice caps at the polar area. In the following figure, a point mass is attached to the surface at 15 degree colatitude and longitude. The coloured dots represent the position where

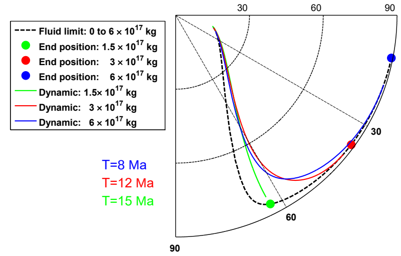

Figure 1: A view of the body in the direction of the rotational axis where the 0 degree longitude line is the direction of the tidal force. The reorientation of each Heaviside type mass anomaly is calculated with time T million years.

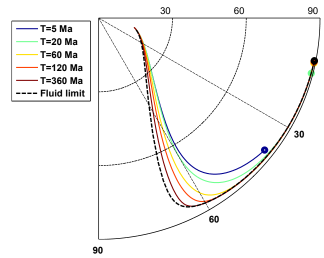

Figure 2: A mass anomaly which increase from 0 to $6 \times 10^{17} kg$ linearly in T million years.

the point mass would end as calculated by the fluid limit solution [*Matsuyama and Nimmo* (2007)]. The coloured lines are the dynamic solutions obtained by our method which suggest a different path. Given a reorientation end position and the history of the loading, we can use our method to constrain the interior viscosities of the model and give a more accurate reorientation path.

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[*Matsuyama and Nimmo* (2007)] Matsuyama, I., and F. Nimmo (2007), Rotational stability of tidally deformed planetary bodies, *Journal of Geophysical Research: Planets*, *112*(E11).