EPSC Abstracts
Vol. 11, EPSC2017-214, 2017
European Planetary Science Congress 2017

True polar wander of slowly rotating object and a case study of Venus

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Abstract

A new semi-analytical method is established to calculate the true polar wander (TPW) of slowly rotating objects such as Venus. Compared with a previous study which is based on the quasi-fluid approximation [*Spada, G. et al* (1996)], a more accurate TPW path is obtained. More importantly, our method can include the coupling effect of the periodic (Chandler wobble) and non-periodic (TPW) terms when the rotation of the body is small. This effect is generally ignored for most of planets and moons such as Earth.

1. Introduction

In the body-fixed frame where the rotational axis aligns with the Z-axis, the linearized Liouville equation shows that the rotational perturbations in the X and Y directions are coupled (m-coupling). This means that in the body-fixed frame, a mass distribution imbalance in the X-Z plane would cause a rotational perturbation not only in the X-Z plane but also in the Y-Z plane. This coupling effect increases as the rotational speed of the object decreases and can turn TPW into a mega-wobble for objects such as Venus which rotate very slowly. Previous studies of TPW for this case applies the quasi-fluid approximation [*Spada, G. et al* (1996)] which can result in an inaccurate TPW path. More importantly, when the rotational speed is slow, the periodic response of the axis which is known as Chandler wobble and the non-periodic TPW is also coupled (p-coupling) and this effect has not been discussed yet.

2. Method

2.1 Moment of Inertia

The change in the inertia tensor for a centrifugally and tidally deformed body is given by

$$\Delta I_{ij}(t) = \frac{k^T(t)a^5}{3G} * [\omega_i(t)\omega_j(t) - \frac{1}{3}\omega^2(t)] + \delta(t) + k^L(t)] * C_{ij}(t) \quad (1)$$

where $\boldsymbol{\omega}$ is the vector of centrifugal force. This equation can be solved analytically by linear change assumption or calculated numerically with a finite-element package. The analytical approach approximates the rotational axis as piecewise linear function with which the equation 1 can be solved analytically. The numerical approach has the potential to include the features such as non-linear rheology or lateral heterogeneity.

2.2 Liouville equation

With the information of the deformation, the reorientation is obtained by combining the perturbation of the rotational and tidal axes. Both can be obtained from a general linearized Liouville equation of non-periodic terms:

$$m_1(t) = \frac{\Delta I_{13}(t)}{C-A} + \frac{C\Delta\ \dot{I}_{23}\ (t)}{\Omega(C-A)^2} \quad (2a)$$

and periodic terms:

$$\bar{m}_1(t) = \frac{1}{(A-C)^2\Omega}$$
$$(\sin\left[\frac{A-C}{A}\Omega t\right]((A-C)\Omega\Delta I_{23} + C\Delta\ \dot{I}_{13}\ (t)) + \quad (3a)$$
$$\cos\left[\frac{A-C}{A}\Omega t\right]((A-C)\Omega\Delta I_{13} + C\Delta\ \dot{I}_{23}\ (t)))$$

Here m_1 and $\bar{m}_1$ are the non-periodic and periodic perturbation in the X-direction, respectively.

2.3 An iterative procedure

When the p-coupling is not considered, an iterative algorithm is applied in each time step [*Hu et al.* (2017)]

Algorithm

1. Assume that the step i starts at time t_i with the vector of the rotation being $\boldsymbol{\omega}^i = \Omega^i(\omega_1^i, \omega_2^i, \omega_3^i)$. and ends at time t_{i+1} with the vector of the rotation $\boldsymbol{\omega}^{i+1}$. For the first iteration, we assume that the vector of the rotation does not change: $\boldsymbol{\omega}^{i+1} = \boldsymbol{\omega}^i$.

2. Obtain $\Delta\mathbf{I}$ and its derivative $\Delta\dot{\mathbf{I}}$ by solving equation 1. With $\mathbf{Q}$ the coordinate transformation matrix from the body-fixed coordinates to the local coordinates where the Z-axis aligns with the direction of the rotation, the inertia tensors in the transformed coordinates are obtained by $\Delta\mathbf{I}_1 = \mathbf{Q}^T\Delta\mathbf{I}\mathbf{Q}$ and $\Delta\dot{\mathbf{I}}_1 = \mathbf{Q}^T\Delta\dot{\mathbf{I}}\mathbf{Q}$.

3. Substitute $\Delta\mathbf{I}_1$ and $\Delta\dot{\mathbf{I}}_1$ into equation 2 and obtain $\boldsymbol{\omega}' = \Omega^i(m_1, m_2, 1+m_3)^T$. We normalize this vector as $\boldsymbol{\omega}' = \Omega^{i+1}\bar{\boldsymbol{\omega}}'$ where $\bar{\boldsymbol{\omega}}'$ is the direction of the perturbed rotational axis in the local coordinate system which needs to be transformed back into the body-fixed frame to obtain $\boldsymbol{\omega}^{i+1} = \Omega^{i+1}\mathbf{Q}\,\bar{\boldsymbol{\omega}}'$ where Ω^{i+1} is the same as in the previous equation.

4. Substitute $\boldsymbol{\omega}^{i+1}$ into step 2 until the result converges.

3. Results

We load the Venus model with a point mass of 1×10^{17} kg which is attached to the surface at 15 degree colatitude and 0 longitude, the rotational speed of the model is set to be 3 times larger than the current day speed of Venus. We compare our method with that of [*Spada, G. et al* (1996)] in figure 1. We simulate the model with increased speed in order to show the difference of the two methods, otherwise the results will be indistinguishable in the plot.

We can see that the result from previous method based on quasi-fluid approximation underestimates the speed of the TPW and gives a different path.
When the p-coupling is also considered, the step-size of our algorithm must be much smaller than the period of the Chandler wobble $\frac{2\pi A}{(C-A)\Omega}$ and equation 3 needs to be added in the iteration.

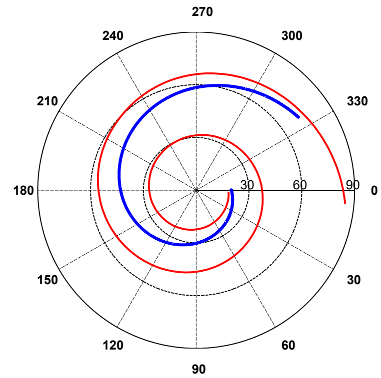

Figure 1: A top-down view of the body where the direction of the rotation is pointing upwards. The blue line is calculated with the quasi-fluid approximation while the red line is from our method

Acknowledgements

This research has been financially supported by the GO program of The Netherlands Organization for Scientific Research (NWO), project number: ALW-GO-12/11.

[*Hu et al.* (2017)] Hu, H., W. van der Wal and B. Vermeersen, A numerical method for reorientation of rotating tidally deformed visco-elastic bodies, *Journal of Geophysical Research: Planets*,2016JE005114.

[*Spada, G. et al* (1996)] Spada, G., Sabadini, R. and Boschi, E., Long-term rotation and mantle dynamics of the Earth, Mars, and Venus, *Journal of Geophysical Research: Planets*, *101*(E1).