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Tidal deformation of Enceladus: variable ice shell thickness

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Abstract

Recent models of Enceladus's interior structure hint at large thickness variations of the ice shell. Here, we investigate the impact of such variations on tidally-induced deformation and stress by means of numerical simulations. We also address a possible scaling of deformation and stress for traditionally used models assuming constant ice shell thickness.

1. Introduction

Joint analysis of Enceladus's low-degree gravity field [1], libration [2] and topography [3] have shown that the Enceladus's ice shell is thin (average thickness $18-26$km) with possibly large thickness variations. Čadek et al. [4] predict the ice shell thickness increasing from few kilometers beneath the south pole to more than 30km in the equatorial area. Such a large thickness variations are expected to influence the response of the ice shell to the tidal forcing. Studies of tidal deformation reflecting non-spherical shape of bodies are nevertheless rather rare [5]. Traditionally used tools and models investigating the tidally induced deformation and stress are usually based on a spectral approach and require spherical bodies with radially dependent material properties. Incorporation of variation of the shell thickness in these models is difficult. For deformation in bodies with an irregular non-spherical shape, an usual choice is to employ a finite element method. Following Souček et al. [6], we have therefore developed a three-dimensional finite element method using FEniCs package [7] which allows to assess the influence of the ice shell thickness variations on the viscoelastic tidal deformation.

2. Model and method

In order to evaluate stress and deformation due to tides, we take into account following equations for a pre-stressed viscoelastic (Maxwell) body:

$$\nabla\cdot\sigma = -\rho\nabla V$$

$$\begin{aligned}\sigma &= \frac{2G(1+\nu)}{3(1-2\nu)}\left(\nabla\cdot\mathbf{u}\right)\mathbf{I} + \\ &\quad G\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\tau} - \frac{2}{3}\left(\nabla\cdot\mathbf{u}\right)\mathbf{I}\right] - \\ &\quad G\int_0^t \frac{\sigma^{\delta}(t')}{\eta}\mathrm{d}t',\end{aligned}$$

where V is the tidal potential, $\mathbf{u}$ is the displacement, σ is the incremental Cauchy stress tensor and σ^{δ} is its deviatoric part. ρ is the ice density, G, ν and η denotes the shear modulus, the Poisson ratio and the viscosity, respectively. The corresponding boundary conditions read

$$\begin{aligned}\sigma\cdot\mathbf{n} + u_{\mathrm{r}}\rho g\mathbf{n} &= \mathbf{0} \quad \text{at surface}\\ \sigma\cdot\mathbf{n} - u_{\mathrm{r}}(\rho_{\mathrm{w}}-\rho)g\mathbf{n} &= -\rho_{\mathrm{w}}V\mathbf{n} \quad \text{at bottom,}\end{aligned}$$

$\mathbf{n}$ is the normal to the boundary, u_r the radial displacement, $\mathbf{g}$ the gravitational acceleration and ρ_{w} is the water density.

The tidal (loading) potential for a body on an eccentric synchronous orbit is described by the following expression [8]

$$\begin{aligned}V &= r^2\omega^2 e\left(-\frac{3}{2}P_{20}(\cos\vartheta)\cos\omega t + \right.\\ &\quad \left.\frac{1}{4}P_{22}(\cos\vartheta)\left(3\cos\omega t\cos 2\varphi + 4\sin\omega t\sin 2\varphi\right)\right),\end{aligned}$$

where t is the time, ω is the angular velocity and e is the eccentricity; P_{jm} are the associated Legendre functions for degree j and order m.

The numerical solution of the governing equations is obtained using FEniCS package (http://fenicsproject.org) [7].

3. Stress and deformation

An example depicting the effect of the ice shell thickness on the tidal deformation and the stress is shown in Fig. 1. Compared to model with the uniform ice shell thickness (25km, model U), the stress is enhanced 6 times in the model with the realistic ice shell thinning (model C) whereas the radial displacement increases only by less than 50%. We also found that the enhancement is degree and order dependent: the potential Love number increases from $k_{2m} = 0.014$ for model U to $k_{20} = 0.017$ and $k_{22} = 0.021$ for model C. The deformation and the additional potential for order $m = 2$ is therefore more enhanced than the order $m = 0$ due to the shell geometry. In the case of model C, the deformation corresponding to the higher degrees ($j = 3-4$) is non-zero with amplitude less than 1% compared to deformation at degree $j = 2$.

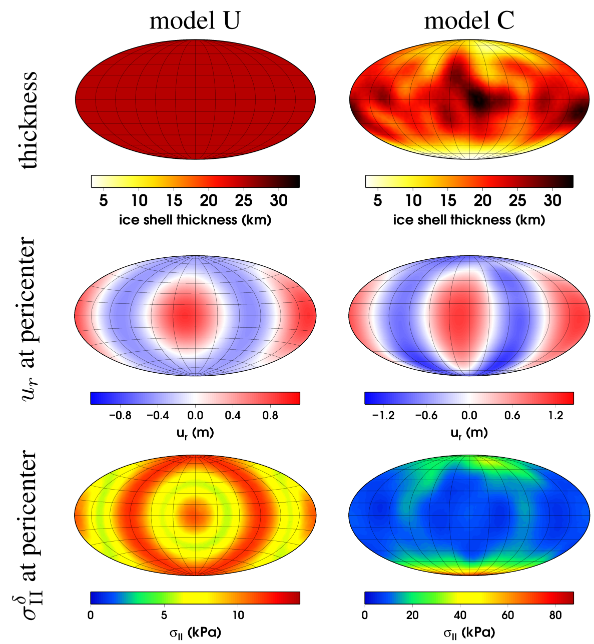

Figure 1: Comparison of displacement and stress at pericenter, models U (uniform thickness, 25km) and model C (based on Čadek et al. [4]), viscosity 10^{14}Pas.

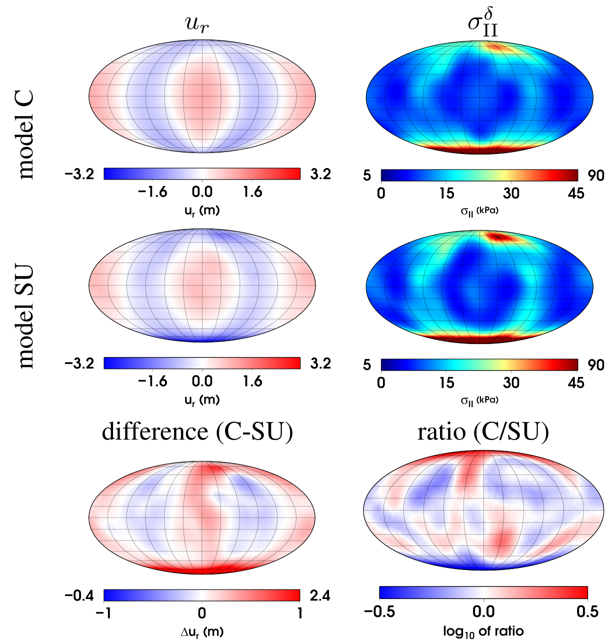

Figure 2: Evaluation of the proposed scaling for displacement and stress, model C and model SU (scaled variables based on model U), elastic body.

4. Scaling

In the case of elastic model, we have addressed the accuracy of the scaling proposed by Turcotte et al. [9]. We have therefore scaled the displacement and stress obtained for the model with the uniform ice shell thickness (model U) by the local ice shell thickness D, i.e. we assume that both radial displacement and stress scales with the factor $25\mathrm{km}/D$. The proposed scaling overestimates both the displacement and the stress magnitude in the southern polar region by the factor ~ 3 (see Fig. 2). Our results therefore suggest necessity to employ fully 3D approach for quantitative assessment of the effects of the thickness variations. The failure of the scaling is probably mainly due to long-wavelength character of the loading which is in breach of the assumptions required for the scaling.

5. Summary and Conclusions

The variable ice shell thickness influences both the magnitude and the pattern of stress and deformation. In the case of the realistic ice shell thickness, the deformation enhancement is degree and order dependent. However, we did not find an appropriate scaling.

Acknowledgements

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