How can periodic orbits puzzle out the coexistence of terrestrial planets with giant eccentric ones?

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Abstract

Hitherto unprecedented detections of exoplanets have been triggered by missions and ground based telescopes. The quest of “exo-Earths” has become intriguing and the long-term stability of planetary orbits is a crucial factor for the biosphere to evolve. Planets in mean-motion resonances (MMRs) prompt the investigation of the dynamics in the framework of the three-body problem, where the families of stable periodic orbits constitute the backbone of stability domains in phase space. In this talk, we address the question of the possible coexistence of terrestrial planets with a giant companion on circular or eccentric orbit and explore the extent of the stability regions, when both the eccentricity of the outer giant planet and the semi-major axis of the inner terrestrial one vary, i.e. we investigate both non-resonant and resonant configurations. The families of periodic orbits in the restricted three-body problem are computed for the $3/2$, $2/1$, $5/2$, $3/1$, $4/1$ and $5/1$ MMRs. We then construct maps of dynamical stability (DS-maps) to identify the boundaries of the stability domains where such a coexistence is allowed. Guided by the periodic orbits, we delve into regular motion in phase space and propose the essential values of the orbital elements, in order for such configurations to survive long time spans and hence, for observations to be complemented or revised.

1. Introduction

Many systems consist of more than one planet and the study of planetary orbits in relation to their long-term stability is crucial. Dynamical studies are essential to determine, whether a given planet can remain stable for long time spans [1]. The sustainability of habitable terrestrial exoplanets, under the effect of another giant planet, whether in resonance, or not, can prove to be a fruitful venture [2, 3].

Using the elliptic restricted three-body problem (ERTBP) as a model, the computation of families of periodic orbits in a suitable rotating frame of reference acts as a diagnostic tool, which can help ascertain information regarding the phase space in planets’ vicinity.

2. Model set-up

We focus on internal MMRs and let two planets of masses $m_1$ and $m_2$, revolve around a star of mass $m_0$, with $m_0 = 1 - m_2$, $m_1 = 0$ and $m_2 = 0.001 = m_J$. We herein restrict our study to coplanar planetary orbits, which correspond to Keplerian ellipses, in the inertial frame, described by heliocentric osculating elements, namely the semimajor axes, $a_i$, the eccentricities, $e_i$ and the longitudes of pericentre, $\varpi_i$. For the position of the planets on the osculating ellipse, we consider the mean anomalies, $M_i$. Subscripts 1 and 2 refer to the inner and the outer planet, respectively. We focus on symmetric periodic orbits, where the apsidal difference is $\Delta \varpi = 0$ or $\pi$.

3. DS-map set-up

DS-maps can depict each neighbourhood in the phase space of a planetary system. By creating grids of initial conditions, after having chosen the periodic orbit being closer to the dynamical vicinity of interest, we colour each condition based on the output of a chaotic indicator and in particular, the DFLI. We perform a thorough study by considering eccentricity values $e_1 = 0.02, 0.1, 0.3$ and 0.5 while $e_2 \in [0, 1]$. Therefore, we showcase the domains where such planetary systems could be hosted.

4. Results

We show the families of periodic orbits that belong to the $3/2$, $2/1$, $5/2$, $3/1$, $4/1$ and $5/1$ MMRs in the ERTBP and then, connect the islands of stability that appear on the DS-maps with the stable periodic orbits. Finally, we present a precise exploration of the phase space
with regards to the extent of those domains when the eccentricities and the angles vary.

In Fig. 1, we depict a particular result taken from one family (blue curve) at 2/1 MMR. It is straightforward that the stable periodic orbits constitute the core of domains where the motion is regular (dark region) and the long-term stability is guaranteed. Within this representative case, we can argue that an inner terrestrial planet can survive long time spans in MMR with a giant highly eccentric outer planet on coplanar symmetric orbits, even when both orbits are highly eccentric.

![Figure 1: A DS-map on the eccentricities' plane, which showcases stable (dark) and chaotic (pale) domains in the phase space of a stable 2/1 resonant symmetric periodic orbit belonging to the family (bold blue curve) with \( \omega_1 = \dot{M}_1 = \pi \) and \( \omega_2 = \dot{M}_2 = 0 \).](image)

5. Summary and conclusions

We study the dynamics of planetary systems consisting of a giant and a terrestrial planet in all major MMRs in the ERTBP and present the phase space in their vicinity. Being motivated by the quest of Earth-like habitable planets we provide all possible stability domains for coplanar symmetric orbits, which are built about stable periodic orbits.

In that respect, this study is essential for the research of terrestrial planets among the systems with giant planets, which have already been discovered and will assist in refining particular observational data.

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References

