

## Asteroid lightcurve inversion with Bayesian inference: Reference phase curves

K. Muinonen (1,2), J. Torppa (3), X. Wang (4), A. Cellino (5), J. Martikainen (1), O. Wilkman (2), M. Gritsevich (1), and A. Penttilä (1)

(1) Department of Physics, University of Helsinki, Finland, (2) Finnish Geospatial Research Institute FGI, Masala, Finland (karri.muinonen@helsinki.fi / Fax: +358-2941-50610), (3) Space Systems Finland, Espoo, Finland, (4) Yunnan Observatories, Kunming, China, (5) INAF, Osservatorio Astrofisico di Torino, Pino Torinese, Italia.

### Abstract

An asteroid's lightcurve, i.e., its observed disk-integrated brightness as a function of time, depends on the shape and spin state of the asteroid, as well as its surface scattering properties. It follows that these properties can be estimated from the observations, to an extent allowed by a given data set. The phase curve of the asteroid refers to the dependence of disk-integrated brightness on the phase angle (the Sun-Object-Observer angle). In the present work, we study reference phase curves that are extrapolated to equatorial illumination and observation of the given asteroid. We show that the reference phase curves, with realistic error bars, can be efficiently derived through statistical lightcurve inversion. These phase curves can have substantial value in asteroid taxonomy.

### 1. Introduction

Lightcurve inversion proceeds conventionally as follows (e.g., [1]). It is reasonable to derive the rotation period with a simplified shape model and a small number of trial pole orientations. Once the period is available, the pole orientation can be refined with a general convex shape model represented by a spherical harmonics expansion for the Gaussian surface density. Once the Gaussian surface density is available, the actual convex shape is constructed as a solution of the Minkowski problem.

In the present work, we devise statistical inverse methods for the retrieval of rotation periods, pole orientations, convex shapes, and scattering properties from the photometric observations. This entails a complete assessment of the uncertainties in the abovementioned physical parameters. We consider conventional photometric data composed of dense lightcurves [1] as well as sparse data mimicking the ongoing observations of the ESA Gaia mission [2]. Our inverse methods comprise both MCMC (Markov

chain Monte Carlo) and importance samplers, building upon the recent advances with the Lommel-Seeliger ellipsoids [3,4], the Monte Carlo virtual-photometry method [5], and an efficient numerical surface scattering model [6].

### 2. Numerical methods

In what follows, we adopt either a triaxial ellipsoid shape model or a general convex shape model, describing the shape by the spherical harmonics expansion for the Gaussian surface density. In the statistical inverse problem, it is our goal to characterize the Bayesian a posteriori probability density for the unknowns. For the ellipsoid model, this involves a small number of unknowns, whereas, for the convex model, this involves high-dimensional forward models with several dozens to hundreds of unknowns. All of the inverse methods are based on the concept of virtual observations, that is, simulated observations obtained by adding random noise to the true observations, and, subsequently, on the virtual least-squares solutions available from the best fits to the virtual observations.

First, in the case of extensive observations, we provide an independence-like importance sampler based on kernel estimation and debiasing of the probability density for the virtual least-squares parameters. Second, in the case of scarce observations, we provide a random-walk MCMC sampler based on the convolution of the probability density for the virtual least-squares parameters by itself, as well as, a random-walk importance sampler based on the same convolved probability density.

### 3. Results and discussion

In Figs. 1 and 2, we highlight the interrelation between conventional asteroid phase curves and reference phase curves in the case of sparse photometry simulated for the Gaia mission. For the

phase curves (Fig. 1), the large variation of the phase curve points is due to the considerably different viewing geometry for the individual phase curve points. The caveat is removed in Fig. 2 by the introduction of the reference viewing geometry: the reference phase curves are smooth from one epoch to another. Finally, we can see that the reference phase curves pertaining to the lightcurve maxima have smaller slopes than the reference phase curves pertaining to the mean brightnesses. In Figs. 1 and 2, the error bars are small and fall inside the plotting symbols.

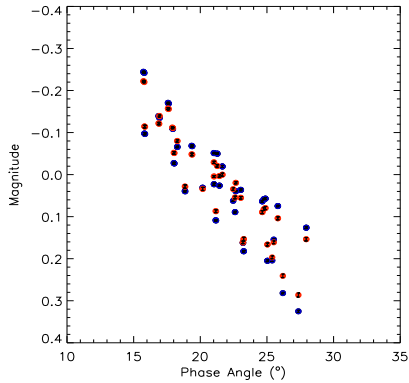


Figure 1: Phase curves corresponding to the mean (blue) and maximum brightness (red) as computed using the sample models at the epoch of each photometric point.

#### 4. Conclusion

We conclude that the present inverse methods promise to settle the issue of missing statistical methods for uncertainty estimation in the lightcurve inversion problem. They allow for efficient retrieval of reference phase curves that can be utilized in asteroid taxonomy. In the future, computational tools based on the present inverse methods will be available through a web-based Gaia Added Value Interface.

#### 5. Acknowledgements

Research supported, in part, by the ERC Advanced Grant 320773.

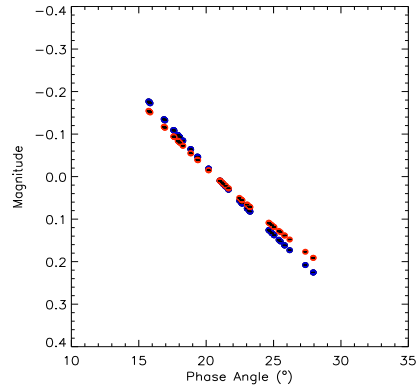


Figure 2: As in Fig. 1 for the reference phase curves.

#### References

- [1] Torppa, J., Hentunen, V-P., Pääkkönen, P., Kehusmaa, P., and Muinonen, K.: Asteroid shape and spin statistics from convex models. *Icarus*, 198, pp. 91-107, 2008.
- [2] Santana-Ros, T., Bartczak, P., Michałowski, T., Tanga, P., and Cellino, A.: Testing the inversion of the asteroids' Gaia photometry combined with ground-based observations. *Mon. Not. Roy. Astron. Soc.*, 450, pp. 333-341, 2015.
- [3] Cellino, A., Muinonen, K., Hestroffer, D., and Carbognani, A.: Inversion of sparse photometric data of asteroids using triaxial ellipsoid shape models and a Lommel-Seeliger scattering law. *Planet. Space Sci.*, 118, pp. 221-226, 2015.
- [4] Muinonen, K., Wilkman, O., Cellino, A., Wang, X., and Wang, Y.: Asteroid lightcurve inversion with Lommel-Seeliger ellipsoids. *Planet. Space Sci.*, 118, pp. 227-241, 2015.
- [5] Wang, X., Muinonen, K., Wang, Y., Behrend, R., Goncalves, R., Oey, J., Antonini, P., Demeautis, C., Manzini, F., Damerdji, J., Montier, J., Klotz, A., Leroy, A., and Ganand, G.: Photometric analysis for the spin and shape parameters of the C-type main-belt asteroids (171) Ophelia and (360) Carlova. *Astron. Astrophysics*, 581, A55, 2015.
- [6] Wilkman, O., Muinonen, K., and Peltoniemi, J.: Photometry of dark atmosphereless planetary bodies: an efficient numerical model. *Planet. Space Sci.*, 118, pp. 250-255, 2015.