

# Tidally driven evolution of differentiated terrestrial exoplanets

M. Walterová and M. Běhounková

Charles University, Faculty of Mathematics and Physics, Department of Geophysics, Czech Republic  
 (kanova@karel.troja.mff.cuni.cz)

## Abstract

The number of confirmed extrasolar planets has already exceeded 3000, providing us with a wide statistical set for the study of their orbital dynamics. In contrast to gas giants, majority of low-mass, possibly terrestrial exoplanets orbits its host star on a circular or only slightly eccentric orbit, which may be a consequence of past or still ongoing tidal evolution. Tidal response has been traditionally described using models based on the assumption of either a constant phase lag or a constant time lag [1, 2], implying however simplified rheology. As these models are not generally applicable to terrestrial moons and planets with realistic properties, recent authors have proposed rheologically motivated analytical [3, 4, 5] or N-body numerical [6] treatment of tides.

Here, we present a numerical model of tidally driven orbital evolution based on the solution of continuum mechanics equations for a differentiated spherical body, whose mantle is governed by either the Maxwell or the Andrade viscoelastic rheology. The model enables generally heterogeneous structure of the mantle, making thus possible the assessment of coupling between the internal and the orbital evolution.

## 1. Model and Methods

Both the long-term orbital evolution and the concurrent despinning or spinning up of the planet is studied using the Gauss's planetary equations complemented with the evolution equation for the rotation rate derived from the conservation of the total angular momentum.

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \left[ eR \sin \nu + \frac{p}{r} S \right], \quad (1)$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{nae} \left[ eR \sin \nu + \left( \frac{p}{r} - \frac{r}{a} \right) S \right], \quad (2)$$

$$\begin{aligned} \frac{d(C\Omega)}{dt} = & -\frac{1}{2} \frac{\mathcal{G}M_*m\sqrt{1-e^2}}{na^2} \frac{da}{dt} \\ & + \frac{M_*m}{M_*+m} \frac{na^2e}{\sqrt{1-e^2}} \frac{de}{dt}. \end{aligned} \quad (3)$$

Here,  $a, e, \Omega$  are the semi-major axis, the orbital eccentricity and the spin rate of the planet, respectively,  $n$  is the mean motion,  $r$  and  $\nu$  indicate the instantaneous star-planet distance and the true anomaly,  $p = a(1-e^2)$  stands for the semilatus rectum of the orbit,  $C$  is the moment of inertia of the planet,  $\mathcal{G}$  the gravitational constant,  $M_*$  and  $m$  the masses of the star and the planet and  $R, S$  are the radial and the perpendicular component of the disturbing force.

The disturbance in the system is caused by tidal deformation of the planet. In order to estimate the boundary deflections, we utilize an extension of the tool and method described in [7, 8], which is based on the numerical solution of governing equations for a viscoelastic continuum,

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

$$-\nabla \pi + \nabla \cdot \mathbf{D} + \mathbf{f} = \mathbf{0}, \quad (5)$$

complemented with the constitutive relation for either the Maxwell or the Andrade rheology. In the equations above,  $\mathbf{u}$  stands for the displacement vector,  $\pi$  and  $\mathbf{D}$  are the isotropic and the deviatoric part of the incremental stress tensor and  $\mathbf{f}$  is the force acting on the planet – in our case a combination of the tidal force, the centrifugal force and the self-gravity. The governing equations are solved in the time domain and discretized using a staggered finite difference scheme in the radial direction and a spherical harmonic decomposition in the lateral directions.

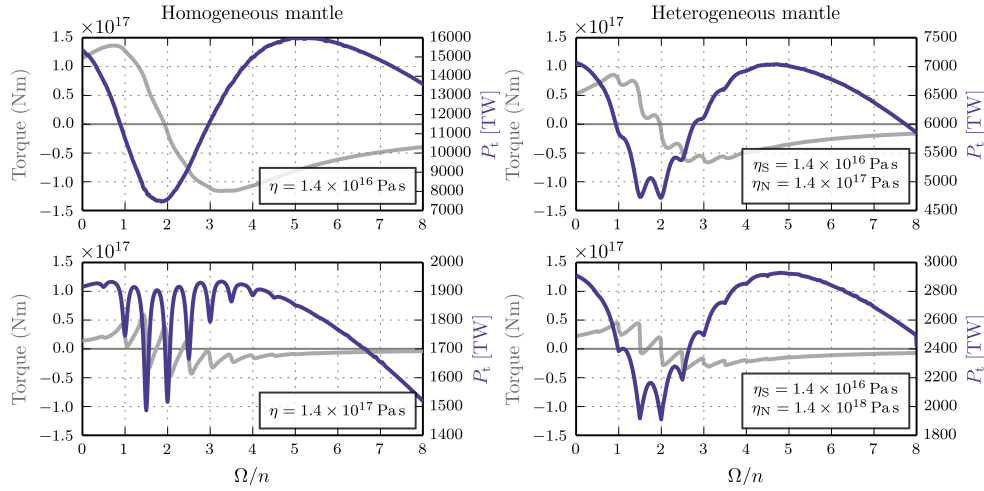


Figure 1: The tidal torque (gray) and the tidal heating (blue) as a function of the loading frequency and the mantle viscosity for a close-in Earth-size terrestrial exoplanet.

## 2. Results

Our model planet consists of a solid inner core, a liquid outer core and a deformable viscoelastic mantle, with possibly heterogeneous viscosity structure. We study the effect of the internal structure on the long-term orbital and rotational evolution, and compare our results with analytical tidal model of homogeneous viscoelastic planet. Furthermore, we compute the rate of the energy dissipation during the orbital evolution and evaluate its effects on the mantle's viscosity.

Figure 1 shows the spin-orbit ratio dependence of the tidal torque and the tidal heating in the case of either homogeneous or heterogeneous structure of the mantle, prescribed as a viscosity difference between the southern and the northern hemisphere. We may see that the energy dissipation inside of a planet with the north-south viscosity structure acquires values different from the results in the case of the homogeneous mantle with average viscosity (lower row). The number and the position of zero points of the tidal torque varies as well, affecting the stability of the corresponding spin states.

## 3. Summary and Conclusions

Numerical computation of tidal evolution may be of great importance when dealing with complex rheologies, planets with non-trivial internal structure or non-spherical shape, and with analytically challenging phenomena. Here, we have presented a method for the evaluation of tides on generally heterogeneous bodies,

enabling for example the analysis of coupling between the orbital and the internal evolution of differentiated terrestrial exoplanets and icy moons.

## Acknowledgements

This work was supported by the Charles University grant SVV (260447/2017) and the IT4Innovations Centre of Excellence project (CZ.1.05/1.1.00/02.0070).

## References

- [1] Kaula, W. M. (1964) *Rev. Geophys.* 2, pp. 661-685.
- [2] Mignard, F. (1979) *Moon and Planets* 20, pp. 301-315.
- [3] Makarov, V. V., Efroimsky, M. (2013) *Astrophys. J.* 764:27.
- [4] Ferraz-Mello, S. (2013) *Celest. Mech. Dyn. Astr.* 116, pp. 109-140.
- [5] Correia, A. C. M., Boué, G., Laskar, J., Rodríguez, A. (2014) *Astron. Astrophys.* 571, A50.
- [6] Frouard, J., Quillen, A. C., Efroimsky, M., Giannella, D. (2016) *Mon. Not. R. Astron. Soc.* 458, p. 2890-2901.
- [7] Tobie, G., Čadek, O., Sotin, C. (2008) *Icarus* 196, pp. 642-652.
- [8] Běhounková, M., Tobie, G., Čadek, O., Choblet, G., Porco, C., Nimmo, F. (2015) *Nat. Geosci.* 8, pp. 601-604.