

A flight of a spaceship along the trajectory “the Earth–the Moon–Mars–the Earth” with one corrective thrust

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1. Introduction

In present, the investigations of the Moon, Mars and small bodies of the Solar system with help of spaceships and artificial satellites with using minimal amount of fuel and maximal mass of scientific techniques with returning at the Earth are of the great interest. Below we are modelling the flight of the interplanetary spaceship with one corrective thrust. The spaceship approaches the Moon and Mars in the gravitational fields of the Sun, the Earth, the Moon and Mars and returns at the Earth. Let's denote - G is gravitational constant, \mathbf{r} is heliocentric radius-vector of the spaceship, \mathbf{r}_E is heliocentric radius-vector of the Earth, \mathbf{r}_{EM} is geocentric radius-vector of the Moon, \mathbf{r}_A is heliocentric radius-vector of Mars, m_S is mass of the Sun, m_E is mass of the Earth, m_M is mass of the Moon, m_A is mass of Mars, v_E is true anomaly of the Earth, $v_M = v_{M0} + kv_E$ is true anomaly of the Moon, $v_A = v_{A0} + mv_E$ is true anomaly of the Mars. Here, k and m are ratios of the orbital angular velocities of the Moon and Mars in respect of the orbital angular velocity of the Earth ω_E . The orbits of the planets are circle and lie in one plane. The independent variable is v_E , coincided in the given planar celestial mechanical model with mean anomaly of the planet (the Earth) and $dv_E/dt = \omega_E$. We are needed to find initial velocity $d\mathbf{v}$ of the spaceship in respect of the Earth, initial positions of the Moon and Mars - v_{A0} and v_{M0} , correspondingly, for which the distances between the ship and the Moon and then between the ship and Mars are minimal and almost are equal to the radii of these natural celestial bodies.

2. The Fundamental equation

The differential equation of the spaceship motion we write in the vector form with one independent variable v_E . The unknown function is \mathbf{r} . So, for the given five body problem we have

$$\begin{aligned} (d^2\mathbf{r}/dv_E^2) \omega_E^2 &= -Gm_S\mathbf{r}/r^3 - \\ Gm_E(\mathbf{r}-\mathbf{r}_E)/|\mathbf{r}-\mathbf{r}_E|^3 - \\ Gm_M(\mathbf{r}-\mathbf{r}_E-\mathbf{r}_{EM})/|\mathbf{r}-\mathbf{r}_E-\mathbf{r}_{EM}|^3 - \\ Gm_A(\mathbf{r}-\mathbf{r}_A)/|\mathbf{r}-\mathbf{r}_A|^3 \end{aligned} \quad (1)$$

3. Example

In a series of numerical evaluations some unknown parameters are found: $dv=11700$ m/s ($d\mathbf{v}$ and the vector of heliocentric velocity of the Earth are oriented perpendicularly to each other; $v_{M0}=0.727$ radians; $v_{A0}=0.69027$ radians. For $v_E=2.861135$ rad the distance between Mars and the spaceship is minimal. (The true (mean) anomaly is measured from the initial position of the vector “the Sun – the Earth”). The minimal distances between the spaceship and the Moon and the spaceship and Mars are equal to 0.000012390865 AU= 1855 km and 0.000034 AU= 5206 km correspondingly. For $v_{A0}=0.6903$ rad the spaceship collides with Mars (the distance between the spaceship and the center of Mars equals 10^{-5} AU). In this case the effect of “the key holes” takes place, e.g. the negligible variations of the initial conditions give the catastrophe [1]. In Fig. 1 the heliocentric trajectory of the spaceship is presented and boundaries of the movement of the spaceship are orbits of Venus and the Main belt of asteroids. Fig. 2 illustrates approaching the spaceship and the Moon. Fig. 3 demonstrates the closing approaching the spaceship and Mars. Fig. 4 shows the value of approaching the spaceship and the Earth after dozens revolutions of the spaceship around the Sun.

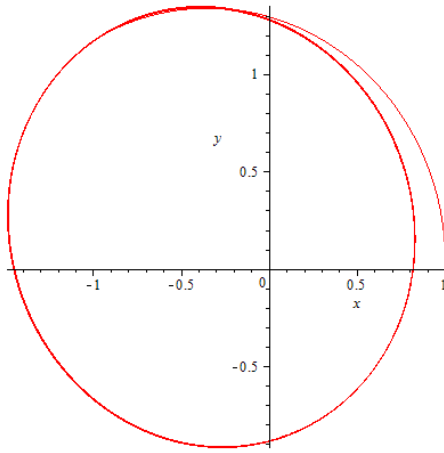


Figure 1: The trajectory of the spaceship in respect of the Sun. The distance is measured in the astronomical units.

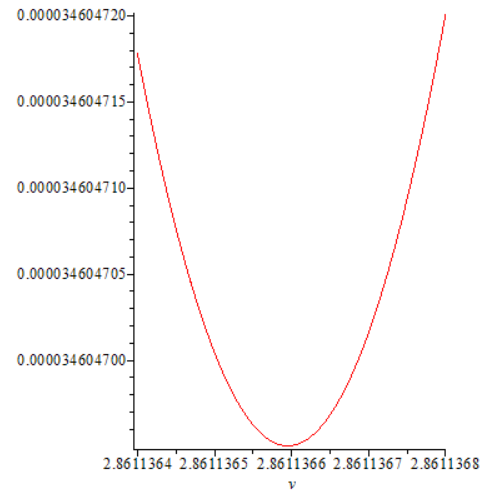


Figure 3: The minimal distance between the spaceship and Mars in astronomical units. v is the true anomaly of the Earth in radians.

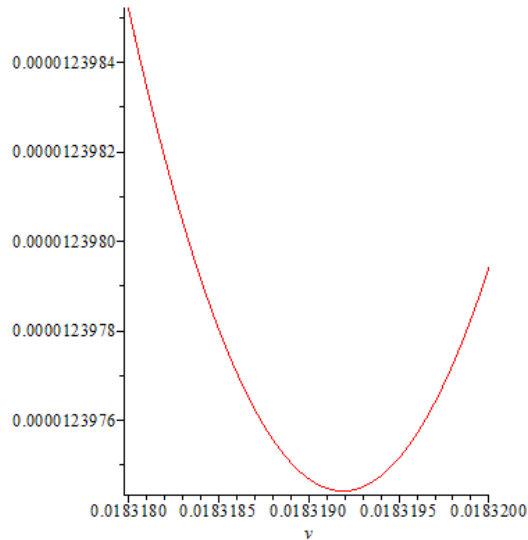


Figure 2: The minimal distance between the spaceship and the Moon is measured in the astronomical units. v is the true anomaly of the Earth is measured in radians.

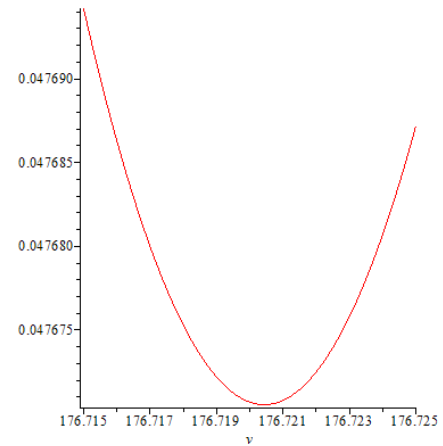


Figure 4: Approaching the spaceship and the Earth. The distance is measured in astronomical units and the true anomaly of the Earth is measured in radians.

4. Conclusions

Searching for the initial conditions for the flights of the interplanetary spaceships with one corrective thrust is refer to the researching method of education and facilitates to organize scientific work of students.

References

- [1] Emel'yanenko, N. Yu.: Features of encounters of small bodies with planets, Solar System Research, Vol. 49, Issue 6, pp.404-409, 2015.

