

## Bias on the “average” photometric behaviour

F. Schmidt (1), S. Bourguignon (2)

(1) GEOPS, Univ. Paris-Sud, CNRS, Université Paris-Saclay, Rue du Belvédère, Bât. 504-509, 91405 Orsay, France (2) Laboratoire des Sciences du Numérique de Nantes, École Centrale de Nantes, France (frederic.schmidt@u-psud.fr)

### Abstract

The Hapke model has been widely used to describe the photometrical behaviour (reflectance as a function of incidence and emergence direction) of planetary surface but the uncertainties of retrieved parameters has been difficult to handle so far. A recent study proposed to estimate the uncertainties using the Bayesian approach [1]. Here, we propose an improvement of the numerical implementation to speed up the uncertainties estimation. We study the common analysis scheme to summarize a collection of data from various locations in order to answer the question: are these locations photometrically homogeneous or not? For instance, this question arises when combining data from an entire planetary body, each pixel with a single *angel* (angular sampling noted *angel*). We tested here the ability of the Bayesian method to decipher two situations, in the presence of noise: (i) a photometrically homogeneous surface (all pixel with the same behaviour), (ii) an heterogeneous surface with 2 distinct photometrical properties (half pixels with behaviour A, other half with behaviour B). The results suggests that the Bayes method is able to distinguish the two situation and thus, to have access to an information about the photometric heterogeneities of a body.

### 1. Introduction

The Bi-directional Reflectance Distribution Function (BRDF) is the core quantity to describe the photometric behaviour [2]. It represents the same location pixel (for picture element), observed with various angular element (angel, for angular element) [3]. Hapke proposed a semi-analytical model of the BRDF of a granular medium [2]. Many authors have been using it to analyze laboratory data [4, 5], telescopic observation [6], in situ data [7], remote sensing data [8] due to its relative simplicity and fast computation. Following our previous study [1], we not to discuss the realism of the photometric Hapke model, but focus on the data analysis point of view in

order to distinguishing homogeneous versus heterogeneous photometric dataset.

### 2. Method

We performed a synthetic test in order to better understand the behaviour of the bayesian analysis in the case of heterogeneous dataset. We generate 100 geometrical configurations randomly with a uniform distribution in an half space in order to define the angels. We then compute the direct Hapke model to create 100 synthetic observations. We propose to consider a noise level of 10% as a upper but realistic bound in this study. We split the 100 angels randomly in two. We attribute the first half with one set of Hapke photometric parameters, namely same single scattering albedo  $\omega$ , roughness angle  $\theta$ , particle phase function parameters  $b/c$  [2]. The other half is attributed to another set of Hapke parameter. This way, we generate a dataset of heterogeneous photometric behaviour (see fig. 1).

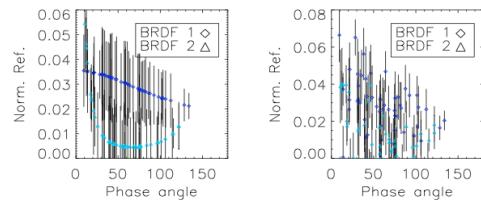


Figure 1: Input data for the synthetic test: normalized reflectance for 100 observations in a random geometry. Hapke parameter are the same  $\omega=0.1$ ,  $\theta=0.5^\circ$ . Dark blue: 50 samples (BRF1) with broad backward scattering ( $b=0.1$ ,  $c=1.0$ ), Light blue: 50 samples (BRF2) with narrow forward scattering ( $b=0.8$ ,  $c=0.1$ ). Left: without noise. Right: with 10% level of noise.

We decide to study one parameter change only, for instance the particle phase function in figure 1, and two choose extreme conditions for this parameter. As

an example, in figure 1 shows broad backward scattering and narrow forward scattering. One can note that the two dataset are clearly separated when there is no noise. When adding noise, it is impossible by eyes to distinguish both datasets. All the game is now to retrieve the Hapke parameters in this heterogeneous configuration that could occur in real dataset, when the observation are not collected at the same time and same location of the surface.

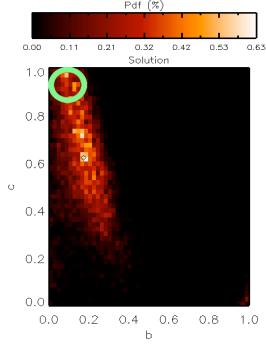


Figure 2: Results of the estimation of  $b$  and  $c$  parameters for 50 observation of BRF1 only. (green circle represent the true solution without noise)

### 3. Results and discussion

Figure 2, 3 and 4 represent the results of the Monte Carlo Bayesian analysis. If we consider only 50 angels of a homogeneous surface (figure 2 and 4), the retrieval method is consistent with the known photometric parameters. Please note that the exact value is not perfectly retrieved simply because of the tolerance due to noise. In the case of 100 angels in a heterogeneous surface (figure 3), the solution is not two maxima near the two true solutions, as one may expect, but an intermediate fake solution. Thus, the Bayesian solution (and obviously all other methods estimating the minimum chi-square solution) seems to be cheated by a heterogeneous dataset.

Nevertheless, an in-depth study of the best solution (minimum chi-square) reveals that only the homogeneous case is compatible with input noise level at a confidence level of 95%. This test fails for the heterogeneous case. Thus, we propose a new approach to distinguish heterogeneous datasets from homogeneous ones, by first using the bayesian

approach to minimize the chi-square without being perturbed by local minima, and then to analyze the confidence of the best solution [9]. This strategy should now be applied in real datasets.

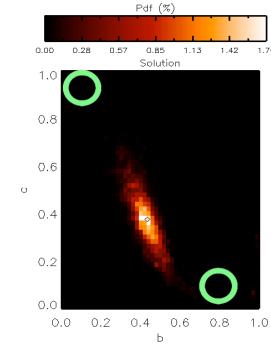


Figure 3: Idem fig. 2 but for 100 observation of BRF1 and BRF2.

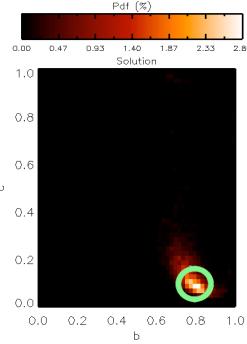


Figure 4: Idem fig. 2 but for 50 observation of BRF2.

### References

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