

Mass and radius of a transiting exoplanet : a Bayesian approach, with application to 55 Cnc-e

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Abstract

Stellar parameters are the key to derive exoplanetary parameters. We derive analytically the joint likelihood of the mass and radius of a host star and its transiting planet, from the probability density functions of the observed quantities. We then use priors on the luminosity, temperature, metallicity and age of the star to constrain more, and estimate better all the stellar parameters. We show that this approach leads to a finer and more correct probability density function of the mass and radius of the star, hence of the planet.

1. Introduction

The discovery of a few thousands of exoplanets has revolutionized planetary science in the last two decades. Now is the time of the characterization of these new worlds. To determine the internal composition of a planet, its mass and radius must be known with great precision. Unfortunately, whereas the ratios of planetary to stellar radii and masses are often known with amazing accuracy, the stellar parameters generally have large uncertainties, often underestimated.

Interferometry offers the possibility of measuring stellar radii with a precision of the order of 1 – 2%. Here, we focus on the 55 Cnc system, because it is one of the few with a transiting exoplanet whose host star is bright enough to be observed by interferometry. Thus, we can provide the joint probability distribution function (PDF) of the mass and radius of the host star, and of the transiting planet 55 Cnc-e.

We then use a Bayesian approach, to benefit from priors on the luminosity, temperature and metallicity of the host star, in order to better constrain all its parameters through stellar evolution models. Finally, we will discuss the influence of these results on the constraints on the planetary structure.

2. Likelihood and priors

2.1. Mass and Radius

55 Cnc has been observed with the VEGA interferometer [5, 2], and its angular diameter has been measured [1, 3] : $\theta = 0.724 \pm 0.012$ mas. From this, one deduces easily the likelihood of its physical radius $R_\star = C\theta/\Pi$ (where $\Pi = 0.08104 \pm 0.00072$ arcsec is the parallax, measured by Hipparcos, and C is a normalization constant, $C = 0.1075 R_\odot$ when θ is in mas and Π in arcsec) :

$$\mathcal{L}_{R_\star}(R) = \frac{C}{R^2} \int_0^{+\infty} f_\theta(t) f_\Pi \left(\frac{Ct}{R} \right) t \, dt. \quad (1)$$

Here, f_X is the PDF of the observed quantity X (assumed Gaussian).

On top of that, the density of the star ρ_\star has been deduced by analysis of the transit light-curve [4]. Its likelihood \mathcal{L}_{ρ_\star} is roughly a Gaussian of mean $1.084 \rho_\odot$ and standard deviation $0.038 \rho_\odot$. These two constraints being independent from each other, in the end the joint likelihood of the mass and radius of 55 Cnc reads :

$$\mathcal{L}^*(M, R) = \mathcal{L}_{R_\star}(R) \times \mathcal{L}_{\rho_\star}(3M/(4\pi R^3)). \quad (2)$$

This is illustrated on the left panel of figure 1.

The right panel of figure 1 displays the joint likelihood of the mass and radius of the transiting planet, which is given by :

$$\mathcal{L}^p(M, R) = \iint \mathcal{L}^*(M_\star, R_\star) \times f_q(M_p/M_\star) \times f_{RR}(R_p/R_\star) \, dM_\star \, dR_\star. \quad (3)$$

Here f_q and f_{RR} represent the PDFs of the planet to star mass ratio and radius ratio respectively.

The mass and radius of the planet are strongly positively correlated, thanks to the estimate of the stellar

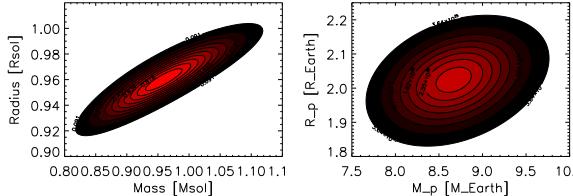


Figure 1: Joint likelihood of the mass and radius of the host star 55 Cnc (left) and the transiting planet 55 Cnc-e (right). The two quantities are clearly correlated, thanks to a constrain on the stellar density (see text).

density. This allows for a more precise estimate of the planet's density than the independent knowledge of its mass and radius would provide. We stress that the above analytic formulas should be used as much as possible.

2.2. Stellar parameters

In the above subsection, we didn't use any prior on the stellar properties. However, stellar evolution models relate a stellar mass, metallicity and age to any other stellar parameter, such as the luminosity, effective temperature, radius, density... Therefore, we can use *a priori* information on 55 Cnc, to exclude unlikely combinations of if mass and radius.

55 Cnc is part of the Hipparcos catalog, for which the number density of stars in the HR diagram is known. As we know its bolometric flux [3], we can derive the joint likelihood of its luminosity and effective temperature. We then multiply this likelihood by our prior : the number density of stars in the HR diagram in the Hipparcos catalog.

Also, 55 Cnc is in the solar neighbourhood for which the distribution of the metallicity is known. Finally, the age of 55 Cnc has been estimated by gyrochronology [4]. Hence, we can give an *a priori* probability to every set of stellar parameters, based on the corresponding metallicity and age.

This Bayesian approach allows us to refine the final estimate of the mass. Figure 2 shows the effect of each prior on the PDF of the mass of the star. It appears that the direct likelihood (bold red line) is significantly modified, although not dramatically changed. This shows that a Bayesian approach is well suited to this problem. In addition, without the use of the priors, one can be slightly off in estimating the final parameters. This is of crucial importance for the characterization of exoplanets.

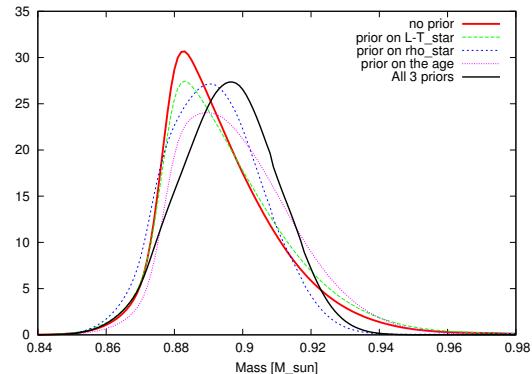


Figure 2: Probability Density Function of the stellar mass, derived from observations and stellar models. The influence of various priors is shown.

3. Conclusion and Perspectives

By combining our Bayesian approach of the stellar parameters and Eq. (3), one gets the most accurate and precise joint probability density function for the mass and radius of the transiting planet 55 Cnc-e. A refined analysis of its internal structure, based on these new estimates, is currently performed. The first results will be presented.

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