

# “Jumping” Trojans

**V.V.Sidorenko**

Keldysh Institute of Applied Mathematics, Moscow, Russia  
(sidorenk@keldysh.ru)

## Abstract

The term “jumping” Trojans is used to specify those asteroids that pass at time to time from the motion around one triangular libration point to another one. We explore the motion of “jumping” Trojans under the scope of the restricted planar elliptical three-body problem. Via double numerical averaging we construct evolutionary equations which allow to analyze the details of the transition between different regimes of the orbital motion.

## Retrospective

In the case of 1:1 mean motion resonance with some of the main planets the asteroid most often moves in “tadpole” orbit (T-orbit in abbreviated form) or in “horseshoe” orbit (HS-orbit). T-orbit cycles around one of the triangular libration points. HS-orbit encompasses both triangular libration points as well as the collinear libration point  $L_3$ . Other types of the resonance coorbital motion are also possible (in particular, quasi-satellite (QS) regimes or compound QS+HS-orbits), but they are less common. The formal difference between these orbits is the behavior of the resonance phase  $\varphi = \lambda - \lambda'$ , where  $\lambda$  and  $\lambda'$  are the mean longitudes of the asteroid and the planet, respectively [6].

If several modes of motion are possible for Hamiltonian system at resonance, then under certain conditions the transitions between them can be observed. In [10] Tsiganis et al. demonstrated that Trojan asteroid (1868) Thersites in the future will make a transition  $T_L \rightarrow T_T$  ( $T_L$  and  $T_T$  denote T-orbits encompassing “leading” libration point  $L_4$  and “trailing” libration point  $L_5$ , respectively). Numerical integration indicates also, that the asteroid 2010TK<sub>7</sub> (the first Trojan asteroid of the Earth) makes transitions between the motions in the neighborhood of  $L_4$  and  $L_5$  [2].

The secular evolution of Trojan asteroids has been studied by many specialists (a detailed bibliography can be found in [4, 8]). Nevertheless, it is likely that only K.Oshima and T.Yahao [7] attempted to investigate theoretically the transitions

$$T_L \rightarrow T_T, T_T \rightarrow T_L, T_{L,T} \rightarrow HS, HS \rightarrow T_{L,T}. \quad (1)$$

Their analysis was based mainly on the consideration of the planar restricted circular three-body problem. Motions with transitions (1) were searched by K.Oshima and T.Yahao in the region of chaotic dynamics generated by the intersection of stable and unstable manifolds of periodic solutions encircling the libration point  $L_3$ . Unfortunately, the interpretation of transitions (1) as a certain homoclinic phenomenon has a serious disadvantage: the measure of the initial conditions giving rise to motions with transitions turns out to be very small ( $\sim e^{-1/\mu^C}$ , where  $\mu$  characterizes the relative part of the planet’s mass in the total mass of the system “Sun+planet”,  $C$  is some positive constant).

## Interpretation of transitions between resonance motions on the base of Wisdom’s approach

Our investigation was undertaken with the aim to demonstrate that in the context of the planar restricted elliptic three-body problem “Sun+planet+asteroid” there exists another mechanism providing the transitions (1). To reveal this mechanism, we apply the basic ideas of the approach proposed by J.Wisdom to study the transformations of the resonance motions [11]. It allows to establish also the dynamical robustness of these transitions in the elliptic problem: they occur for the set of initial conditions which measure does not depend on  $\mu$ . Previously we studied in a similar way the formation and the destruction of the QS regimes of the orbital motion [9].

Technically, the Wisdom's approach is reduced to two averaging of the motion equations. The first averaging is carried out over the orbital motion, whereafter the phase variables are rescaled, and the problem is shaped into a form called a "slow-fast" system (SF-system). This is a two degrees of freedom Hamiltonian system with the variables evolving at different rates: some variables are "slow", while the other are "fast". The second averaging is then performed over the "fast" motions of the SF-system. This provides us the evolutionary equations describing the secular effects in the asteroid's motion.

## Summary

We hope that our analysis will be a useful addition to previous studies of the secular effects in the dynamics of Trojan asteroids on the basis of the modern theory of resonance phenomena in Hamiltonian systems [1,5]. Considering the three-body problem, we can't explain the transition  $T_L \rightarrow T_T$  demonstrated by the asteroid (1868) Thersites: the numerical results presented in [10], indicate a significant influence of secular resonances on the dynamics of this asteroid. The mechanism of transitions that we are discussing is probably realized in the dynamics of the so called "temporary" Trojans [3]. Since their stay in certain regime of motion is relatively short, the effects due to secular resonances might be neglected.

## Acknowledgements

Our work was supported by the Presidium of the Russian Academy of Sciences under the scope of the Program 7 "Fundamental problem of Solar system investigations".

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