

# Compressible MHD turbulence in the Earth's magnetosheath: estimation of the energy cascade rate using *in-situ* spacecraft data

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## Abstract

Using the exact law of compressible isothermal magnetohydrodynamic (MHD) turbulence, we give the first estimation of the energy cascade rate ( $|\epsilon|$ ) in the Earth's magnetosheath using THEMIS and CLUSTER spacecraft data. We show that  $|\epsilon|$  is at least three orders of magnitude larger than its value in the solar wind. We identify different type of turbulent fluctuations (magnetosonic and Alfvénic-like) with different properties and scaling laws relating the turbulent Mach number and the energy cascade rate. This observational study can actually help improving current models of astrophysical turbulence by addressing the role of compressibility behind astrophysical shocks, in the interstellar medium or in supernova remanents. This work is currently in preparation for submission [3].

## 1. Introduction

Compressible turbulence has been a subject of active research within the space physics community for the last three decades especially that it is believed to be essential for understanding the physics of the solar wind (for instance the heating of the fast wind), of the interstellar medium (in cold molecular clouds) and other astrophysical and space phenomena. Since the magnetosheath is characterized by a high level of density fluctuations,  $\sim 50\% - 100\%$  [5, 2] in comparison with  $5\% - 20\%$  in the solar wind, it actually represents a key region of the near-Earth space where significant progress can be made in understanding compressible plasma turbulence, which is poorly modeled or understood.

### 1.1. Compressible and incompressible exact laws in MHD

The role of density fluctuations is highlighted by comparing the results obtained from the exact laws of MHD isothermal compressible model (BG13) derived recently by Banerjee and Galtier [1] and the incompressible MHD model derived by Politano and Pouquet [4] (PP98). Under the assumptions of time stationarity, space homogeneity and isotropy turbulence, the PP98 exact law is given by:

$$-\frac{4}{3}\epsilon_I \ell = \left\langle \frac{(\delta \mathbf{z}^+)^2}{2} \delta z_\ell^- + \frac{(\delta \mathbf{z}^-)^2}{2} \delta z_\ell^+ \right\rangle \rho_0, \quad (1)$$

and the BG13 model is given by

$$\begin{aligned} -\frac{4}{3}\epsilon_C \ell &= \left\langle \frac{1}{2} [\delta(\rho \mathbf{z}^-) \cdot \delta \mathbf{z}^+] \delta z_\ell^+ + \frac{1}{2} [\delta(\rho \mathbf{z}^+) \cdot \delta \mathbf{z}^-] \delta z_\ell^- \right\rangle \\ &+ \langle 2\delta\rho\delta e\delta v_\ell \rangle \\ &+ \left\langle 2\bar{\delta} \left[ \left(1 + \frac{1}{\beta}\right) e + \frac{v_A^2}{2} \right] \delta(\rho_1 v_\ell) \right\rangle \end{aligned} \quad (2)$$

where  $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{v}_A$  represents the Elsässer variables,  $\mathbf{v}$  being the plasma flow velocity,  $\mathbf{v}_A \equiv \mathbf{B}/\sqrt{\mu_0 \rho_0}$  is the magnetic field normalized to a velocity and  $\rho_0 = \langle \rho \rangle$  the mean plasma density,  $\delta \mathbf{z}^\pm \equiv \mathbf{z}^\pm(\mathbf{x} + \ell) - \mathbf{z}^\pm(\mathbf{x})$  is the spatial increment of  $\mathbf{z}^\pm$  at a scale  $\ell$  in the radial direction,  $\langle \dots \rangle$  is the ensemble average,  $\bar{\delta}\psi \equiv (\psi(\mathbf{x} + \ell) + \psi(\mathbf{x}))/2$ ,  $e = c_s^2 \ln(\rho/\rho_0)$  is the internal energy, with  $c_s$  the constant isothermal sound speed,  $\rho$  the local plasma density ( $\rho = \rho_0 + \rho_1$ ) and  $\beta = 2c_s^2/v_A^2$  is the local ratio of the total thermal to magnetic pressure ( $\beta = \beta_e + \beta_p$ ).

## 2. Observations and results

We use the *in situ* wave and plasma data from the Cluster and Themis spacecraft.

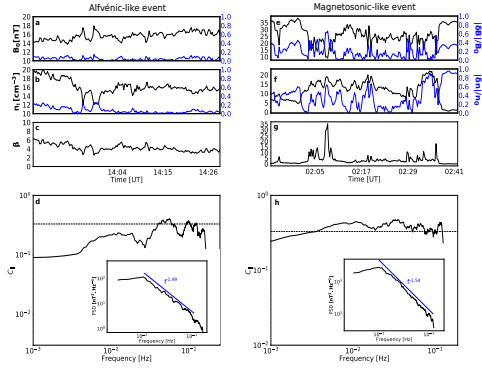


Figure 1: (Right) Incompressible Alfvénic and (left) compressible magnetosonic events. (d) and (h) the corresponding magnetic compressibility.

Figure 1 shows two examples of the analyzed group of data. An incompressible Alfvénic case study and a compressible magnetosonic one. This was done using the magnetic compressibility  $C_{\parallel} = \delta B_{\parallel}^2 / \delta B^2$  (i.e., the ratio between the PSDs of the parallel to the total magnetic fluctuations; parallel being along the mean background field  $B_0$ ).

Figure 2 shows the corresponding cascade rates,  $|\epsilon_C|$  and  $|\epsilon_I|$ , from the compressible BG13 and the incompressible model PP98, respectively. Two main observations can be made: first, the incompressible cascade rate  $|\epsilon_I|$  is larger by a factor  $\sim 100$  in the magnetosonic case compared to the Alfvénic one. Second, density fluctuations in the magnetosonic case amplify

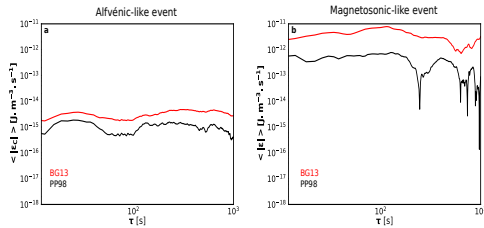


Figure 2: The energy cascade rates computed using BG13 (red) and PP98 (black) for the same (a) Alfvénic and (b) magnetosonic-like events of Figure 1.

$|\epsilon_C|$  by a factor  $\sim 7$  w.r.t.  $|\epsilon_I|$ . These observations are representative of the all the samples (not shown here).

## 3. Summary and Conclusions

This study has provided the first estimation of  $|\epsilon|$  in MHD turbulence in a the compressible magnetosheath plasma. Furthermore, other features related to the turbulent Mach number were identified (not shown here).

## References

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