

# Geodetic Framework for Martian Satellite Exploration I: Reference Rotation Models

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## Abstract

We study most recent orbital data of Phobos and Deimos to derive reference rotational parameters of the two Martian satellites. With the two Mars companions captured in a spin-orbit resonance, their period of rotation is coupled to the orbital motion and the rotation axis follows the long-term precession and nutation of the orbit plane normal caused by perturbations by the Sun and planets. A comparison of our reference rotation parameters with actual measurements of the satellites' rotations may shed light on interior states and structure parameters.

## 1. Introduction

The two Martian moons, Phobos and Deimos, orbit deep in the gravity field of their parent body. Tidal forces acting on the irregular shapes of the satellites have caused a rotational state which is resonant to their orbital motion. Due to the spherical asymmetry of the Martian gravity field as well as third-body perturbations by the Sun and the planets the orbits of the satellites deviate remarkably from perfect Keplerian orbits. For instance, the orbit plane normal as well as the argument of pericenter performs a precessional motion with main components of 2.26 and 54.5 years for Phobos and Deimos, respectively. The rotational states of the satellites follow adiabatically these long-period changes of the orbits and remain trapped in their spin-orbit resonance. Through a precise analysis of the orbital motion of the satellites and the condition of a synchronous rotation we can derive their resonant rotation states and use them for interpretation to actual measurements of their rotation [6].

## 2. Rotation of Phobos and Deimos

While the Martian satellites were discovered in 1877, it was the Mariner 9 spacecraft, which provided the first close-up observation of Phobos and Deimos. At that time, first rotational models based on the orbital motion were devised [1]. Based on Viking Orbiter

data, a control network analysis was performed [2] providing the first observation of a forced libration of Phobos. These measurements were recently updated by Jacobson et al., [3], Willner et al., [4], Oberst et al., [5] and Burmeister et al., [6]. Likewise the model for the orbital motion of the Martian moons has been updated upon availability of new astrometric observations. In contrast the parameters for the resonant rotational state of the satellites remained the same throughout the decades leading to a disagreement between the rotation models for Phobos and Deimos and their actual state. Using most recent solutions for the orbits of the satellites we want to compute the resonant rotational states including the forcing terms for the physical librations in longitude.

## 3. Data and Method

The latest Martian moon ephemerides (mar097, [3]) covering a period of 200 years centered at the year 2000 were used for this analysis. With a sampling rate of 21.6 minutes we first compute the osculating orbital elements of the satellite in an inertial reference frame, i.e. International Celestial Reference Frame (ICRF). The resulting time series is then decomposed in a secular trend and a sum of periodic terms using an iterative algorithm. At first a Fourier transformation of the signal (time series of one osculating orbital element) is performed to identify the frequency of the highest amplitude in the power spectrum. Secondly a windowed Fourier transform at the identified frequency is used to constrain the frequency, phase, and amplitude of the signal component more precisely. The derived values were used as initial values within a least-squares fit of the secular and periodic parts. The obtained fit parameters are then used to extract the identified component from the original signal and the iteration cycle starts again with the residuals as input signal. More details on this approach can be found in [7].

As we want to derive resonant rotational parameters we can directly infer the orientation of the rotation axis (declination  $\delta$  and right ascension  $\alpha$ ) from the inclination of the orbital plane and the

longitude of the ascending node. The rotation angle  $W$  is derived from the sum of mean anomaly and argument of pericenter.

## 4. Results

The resonant rotation parameters for Phobos are

$$\begin{aligned} \alpha &= 317.67071657^\circ - 0.10844326^\circ T \\ &\quad -1.78428399^\circ \sin(M1) + 0.02212824^\circ \sin(M2) \\ &\quad -0.01028251^\circ \sin(M3) - 0.00475595^\circ \sin(M4) \\ \delta &= 52.88627266^\circ - 0.06134706^\circ T \\ &\quad -1.07516537^\circ \cos(M1) + 0.00668626^\circ \cos(M2) \\ &\quad -0.00648740^\circ \cos(M3) + 0.00281576^\circ \cos(M4) \end{aligned}$$

$$\begin{aligned} W &= 35.18774440^\circ + 1128.84475928^\circ d + 12.72192797^\circ T^2 \\ &\quad + 1.42421769^\circ \sin(M1) - 0.02273783^\circ \sin(M2) \\ &\quad + 0.00410711^\circ \sin(M3) + 0.00631964^\circ \sin(M4). \end{aligned}$$

The forcing terms of the physical libration in longitude of Phobos are given by

$$\begin{aligned} W_{\text{lib}} &= 1.73203319^\circ \sin(M5) + 0.01635407^\circ \sin(M6) \\ &\quad + 0.00021426^\circ \sin(M7) \end{aligned}$$

and the arguments of the trigonometric functions are

$$\begin{aligned} M1 &= 190.72646643^\circ + 15917.10818695^\circ T \\ M2 &= 21.46892470^\circ + 31834.27934054^\circ T \\ M3 &= 332.86082793^\circ + 19139.89694742^\circ T \\ M4 &= 394.93256437^\circ + 38280.79631835^\circ T \\ M5 &= 189.63271560^\circ + 41215158.18420050^\circ T \\ &\quad + 12.71192322^\circ T^2 \\ M6 &= 19.26538605^\circ + 82430316.36864280^\circ T \\ &\quad + 25.42412173^\circ T^2 \\ M7 &= 208.89882434^\circ + 123645474.54466790^\circ T \\ &\quad + 38.13293168^\circ T^2. \end{aligned}$$

The resonant rotation parameters for Deimos are

$$\begin{aligned} \alpha &= 316.65705808^\circ - 0.10518014^\circ T \\ &\quad + 3.09217726^\circ \sin(M8) + 0.22980637^\circ \sin(M9) \\ &\quad + 0.06418655^\circ \sin(M10) + 0.02533537^\circ \sin(M11) \\ &\quad + 0.00778695^\circ \sin(M12) \\ \delta &= 53.50992033^\circ - 0.05979094^\circ T \\ &\quad + 1.83936004^\circ \cos(M8) + 0.14325320^\circ \cos(M9) \\ &\quad + 0.01911409^\circ \cos(M10) - 0.01482590^\circ \cos(M11) \\ &\quad + 0.00192430^\circ \cos(M12) \\ W &= 79.39932954^\circ + 285.16188899^\circ d \\ &\quad - 2.73954829^\circ \sin(M8) - 0.39968606^\circ \sin(M9) \\ &\quad - 0.06563259^\circ \sin(M10) - 0.02912940^\circ \sin(M11) \\ &\quad + 0.01699160^\circ \sin(M12). \end{aligned}$$

The forcing terms of the physical libration in longitude of Deimos are given by

$$\begin{aligned} W_{\text{lib}} &= 0.03080596^\circ \sin(M13) \\ &\quad + 0.01248044^\circ \sin(M14) - 0.00437509^\circ \sin(M15) \end{aligned}$$

and the arguments of the trigonometric functions are

$$\begin{aligned} M8 &= 121.46893664^\circ + 660.22803474^\circ T \\ M9 &= 231.05028581^\circ + 660.99123540^\circ T \\ M10 &= 251.37314025^\circ + 1320.50145245^\circ T \\ M11 &= 217.98635955^\circ + 38279.96125550^\circ T \\ M12 &= 196.19729402^\circ + 19139.83628608^\circ T \\ M13 &= 10.80071482^\circ + 10414879.22849759^\circ T \\ M14 &= 345.99306351^\circ + 4801583.39793913^\circ T \\ M15 &= 303.28024985^\circ + 10415546.40050500^\circ T, \end{aligned}$$

with  $d$  and  $T$  denoting the ephemeris time measured in days and centuries, respectively.

The value for the prime meridian constant (zero order term in the expression for  $W$ ) represents the orientation of an averaged rotation of the satellite at the J2000 epoch. Thus, the prime meridian of the satellite is defined dynamically, i.e. based on the orbital motion. This ensures that at apo- and pericenter the x-axis of the satellites body-fixed frame coincides with the direction to Mars' center of mass (within the accuracy of the rotation model of about 40 arc second).

In order to incorporate the physical libration in longitude into the rotational model of Phobos the expression for  $W$  has to be modified according to

$W \rightarrow W + A1 \sin(M5) + A2 \sin(M6) + A3 \sin(M7)$ , where  $A1$ ,  $A2$  and  $A3$  are measurements of the forced libration amplitudes at the corresponding frequencies of the forcing terms in  $W_{\text{lib}}$ .

## 5. Discussion

The need for updated resonant rotational parameters for Phobos and Deimos was stated by Thomas C. Duxbury. He noticed a significant 1.4 degree deviation of Phobos' x-axis orientation with respect to Mars' center of mass, when rotational parameters currently adopted for Phobos are used [8]. This observation sparked discussion among the geodetic community. Robert A. Jacobson released a similar set of parameters for the rotation of the Martian moons that is equal to the presented model within about 40 arc seconds (about 5 m at Phobos surface) [9]. The current accuracy is considered sufficient for cartographic purposes but might need improvement for tasks like high precision landing on Phobos.

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## References

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