

# Planetary Rings: Deviation from Energy-Equipartition the Temperature-Vector

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## Abstract

Planetary rings are ensembles of granular (icy) aggregates ranging in size from centimetres up to a few metres. They form an extremely thin Keplerian disk – vertical extent of about 10 metres – driven by a steady shear caused by the gravity of the central planet. The ensemble is dominated by dissipative collisions which, in densest regions of Saturn’s rings, reach more than  $> (3\Omega\tau)^{-1}$  collisions per orbital period  $T = 2\pi/\Omega$  (Kepler-orbital frequency  $\Omega$ ). The optical depth  $\tau \propto \sigma$  is a measure for the surface-mass density  $\sigma$  in the rings. Each collision is able to either dissipate thermal energy of the ring-aggregates while it may change its size due to aggregation or fragmentation [1]. A balance between aggregation and fragmentation has found to successfully explain the observed size distribution of e.g. Saturn’s rings [2] under assumptions of Maxwellian velocity distribution (VD) and energy-equipartition in form of an unique granular temperature  $T$  characteristic for all particles sizes  $k$  ( $k$  – number of model-monomers the aggregated consist of).

However, an expression of the deviation from the thermodynamic equilibrium of the ensemble – irreversible collisional processes dissipate kinetic (thermal) energy of the ensemble – is the violation of the energy equipartition. The latter is characteristic for conservative Hamiltonian systems.

In this work we quantify this effect by describing a balance between granular cooling  $\propto (1 - \epsilon^2)$  and viscous heating  $\propto \nu\Omega^2$ , again under the assumption of Gaussian VD, but with *mass dependent* granular temperatures  $T(k) = T_k$ . Here, the restitution coefficient  $\epsilon$  (ratio of the normal impact speed before and after the collision) is assumed to be rather small  $\epsilon \ll 1$  and constant, while  $\nu$  labels the granular viscosity. Using the momentum-conservation and the energy-balance at a single binary collision between a small (mass  $m_s$ ) and a large (mass  $m_l$ ) we show that the respective specific energy-dissipation  $\Delta E_{s/l} \propto m_{s/l}/(m_s + m_l)$  is the larger the smaller the ring-aggregate is. In other

words, the deviation from the energy equipartition becomes largest for the smallest members of the aggregate ensemble. We quantify this effect in a steady state by using the mass/size distribution  $n_k = n(k)$  derived by Brilliantov et al. [2].

The extension of the state variables a temperature vector  $T_k$ , in addition to mass-densities  $\rho_k$  and velocities  $\vec{u}_k \approx \vec{u}$ , allows to characterize the mass/size dependence of transport-processes (e.g.  $\nu \rightarrow \nu_k$ ). This offers the chance to investigate possible related instabilities like e.g. mass segregation, clustering.

## References

- [1] Spahn, F., Albers, N., Sremcevic, M., and Thornton, C. 2004. Kinetic description of coagulation and fragmentation in dilute granular particle ensembles. *Europhys. Lett.*, **67**, 545–551.
- [2] Brilliantov, N. V., Krapivsky, P. L., Bodrova, A., Spahn, F., Hayakawa, H., Stadnichuk, V., and Schmidt, J. 2015. Size distribution of particles in Saturn s rings from aggregation and fragmentation. *PNAS*, **112**, 9536–9541.