

Planetary Rings: Deviation from Energy-Equipartition the Temperature-Vector

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Abstract

Planetary rings are ensembles of granular (icy) aggregates ranging in size from centimetres up to a few metres. They form an extremely thin Keplerian disk – vertical extent of about 10 metres – driven by a steady shear caused by the gravity of the central planet. The ensemble is dominated by dissipative collisions which, in densest regions of Saturn’s rings, reach more than $> (3\Omega\tau)^{-1}$ collisions per orbital period $T = 2\pi/\Omega$ (Kepler-orbital frequency Ω). The optical depth $\tau \propto \sigma$ is a measure for the surface-mass density σ in the rings. Each collision is able to either dissipate thermal energy of the ring-aggregates while it may change its size due to aggregation or fragmentation [1]. A balance between aggregation and fragmentation has found to successfully explain the observed size distribution of e.g. Saturn’s rings [2] under assumptions of Maxwellian velocity distribution (VD) and energy-equipartition in form of an unique granular temperature T characteristic for all particles sizes k (k – number of model-monomers the aggregated consist of).

However, an expression of the deviation from the thermodynamic equilibrium of the ensemble – irreversible collisional processes dissipate kinetic (thermal) energy of the ensemble – is the violation of the energy equipartition. The latter is characteristic for conservative Hamiltonian systems.

In this work we quantify this effect by describing a balance between granular cooling $\propto (1 - \epsilon^2)$ and viscous heating $\propto \nu\Omega^2$, again under the assumption of Gaussian VD, but with *mass dependent* granular temperatures $T(k) = T_k$. Here, the restitution coefficient ϵ (ratio of the normal impact speed before and after the collision) is assumed to be rather small $\epsilon \ll 1$ and constant, while ν labels the granular viscosity. Using the momentum-conservation and the energy-balance at a single binary collision between a small (mass m_s) and a large (mass m_l) we show that the respective specific energy-dissipation $\Delta E_{s/l} \propto m_{s/l}/(m_s + m_l)$ is the larger the smaller the ring-aggregate is. In other

words, the deviation from the energy equipartition becomes largest for the smallest members of the aggregate ensemble. We quantify this effect in a steady state by using the mass/size distribution $n_k = n(k)$ derived by Brilliantov et al. [2].

The extension of the state variables a temperature vector T_k , in addition to mass-densities ρ_k and velocities $\vec{u}_k \approx \vec{u}$, allows to characterize the mass/size dependence of transport-processes (e.g. $\nu \rightarrow \nu_k$). This offers the chance to investigate possible related instabilities like e.g. mass segregation, clustering.

References

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- [2] Brilliantov, N. V., Krapivsky, P. L., Bodrova, A., Spahn, F., Hayakawa, H., Stadnichuk, V., and Schmidt, J. 2015. Size distribution of particles in Saturn s rings from aggregation and fragmentation. *PNAS*, **112**, 9536–9541.