

Asteroid taxonomy with limited spectral ranges

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1. Introduction

Asteroid taxonomic systems are classification schemes for asteroids based on their spectrophotometric properties. For the typical UV-vis-NIR wavelengths, the photometric observations probe the surface (regolith) properties of the target. As the spectral slope and the absorption bands are resulting from the light scattering mechanisms, mainly absorption, in the regolith material, the spectra contains information on the absorption coefficient of the material. Furthermore, by comparing the spectral behavior to laboratory measurements of different materials, we can find the possible mineral composition of the target.

There are several taxonomic classification systems, often created after new asteroid spectrophotometric survey program when there is novel observational data available. Some most popular examples are the Tholen taxonomy [1] using observations from the Eight-Color Asteroid Survey (ECAS), the Bus taxonomy [2] using the Small Main-Belt Asteroid Spectroscopic Survey (SMASS), and the Bus-DeMeo taxonomy [3] combining the SMASS II data with NIR observations with the SpeX instrument at the NASA infrared telescope facility. In the future, the Gaia space telescope will produce a huge data set of asteroid spectra in UV-vis-NIR range, and will most probably also produce its own taxonomic classification.

The problem with the different taxonomic systems with new observations is that they are build on the wavelengths used in their observational data set. Tholen uses eight wavelengths between 0.31 and 1.06 μm and the albedo. Bus uses wavelengths from 0.44 to 0.92 μm and with spectral resolution much higher than in ECAS. Bus-DeMeo employ wavelengths 0.45–2.45 μm , and the Gaia data will have wavelengths 0.33–1.05 μm . Now, how to classify an object if your observed spectra is not exactly from these wavelengths?

I propose here a method that is suitable for finding the Bus-DeMeo taxonomic classification for objects with observations from a spectral range that is a subset of the nominal 0.45–2.45 μm range. The method is

based on the linear discriminant analysis of the original Bus-DeMeo data set of 371 asteroids that is tuned every time for the specific wavelength range, and on the Naïve Bayesian Classifier technique.

2. Spectral classifier for Bus-DeMeo taxonomy with a limited spectral range

The purpose of the method described below is to associate the spectral observation of a given target with probabilities for Bus-DeMeo (B-DM) taxonomic classes. The condition for the observed spectra is, that it is a subset of the spectral range of the original B-DM classification, that is, 0.45–2.45 μm .

For every set of spectral data with the same wavelength range ($\lambda_{low}, \lambda_{high}$), the original B-DM data of 371 asteroids is converted on-the-fly with the linear discriminant analysis (LDA) transform using the same wavelength range for the data.

The LDA is a method closely related to the principal component analysis (PCA), but with the ability for finding a new coordinate basis so that the differences *between the known classes* are maximized, whereas in PCA the classes in the data are not known beforehand. With the centered (i.e., wavelength channel means \mathbf{m} subtracted) data matrix \mathbf{X} , its covariance matrix \mathbf{S} , and group-wise covariance matrices \mathbf{S}_c , the LDA is formed using the within-group and between-groups covariance matrices \mathbf{W} and \mathbf{B} as

$$\begin{aligned}\mathbf{W} &= \sum_c (n_c - 1) \mathbf{S}_c, \\ \mathbf{B} &= \sum_c n_c (\mathbf{m}_c^T \mathbf{m}_c),\end{aligned}\quad (1)$$

where c iterates over the groups, \mathbf{m}_c is the channel mean vector for group c , and n_c is the number of targets in group c .

From these the LDA projection matrix is constructed with the help of the eigenvalue decomposition,

$$\mathbf{LAL}^{-1} = \mathbf{W}^{-1}\mathbf{B},\quad (2)$$

Now the matrix \mathbf{L} holds the eigenvectors that project the spectral values to LDA-space. The eigenvalue magnitudes order the projections from the most relevant to the least relevant, and only the few first dimensions are needed. Finally, the LDA-projected data (both the original B-DM targets and the observations to be classified) is received with the transformation

$$\mathbf{y} = (\mathbf{x} - \mathbf{m})\mathbf{L}. \quad (3)$$

The actual classifier, the naïve Bayesian classifier (NBC), is using the LDA-transformed B-DM data (here, \mathbf{Z}) as a learning set to classify the new observations \mathbf{y} . The NBC treats every class as a (Gaussian) probability distribution, and our training set gives the estimated parameters to these distributions. The NBC probability for an observation \mathbf{y} to belong to class c is therefore computed as

$$p_c = a_c f(\mathbf{y}; \mathbf{m}_c^*, \mathbf{S}_c^*), \quad (4)$$

where f is the probability density function of multivariate Gaussian distribution, and a_c is the a priori probability of class c . The NBC probability will be computed for all the classes, and the most probable class is selected.

3. Discussion

I present here a method to relate asteroid spectral observations to the Bus-DeMeo taxonomy, even if the wavelength range of the observed spectra does not meet the range of the Bus-DeMeo system. I have selected the Bus-DeMeo taxonomy here because it offers the widest wavelength range to be used. We have tested this method in a special case with the Gaia wavelengths [4], but it can be used with all Bus-DeMeo wavelength range subsets.

References

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