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Spin-orbit resonances around an ringed elongated body: beyond the first order

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Abstract

Mean motion resonances between ring particles and a perturbing satellite have been widely studied in the case of the giant planets. Among them, the Lindblad first-order resonances cause spiral-wave patterns in the ring, with an exchange of angular momentum between the disk and the satellite. Higher-order resonances are weaker and usually cause the self-crossing of the resonant streamlines, and have been little studied for those reasons. The recent discoveries of rings around small and non-spherical bodies like Chariklo and Haumea [1, 2] have triggered new interests, as the large elongations of the bodies cause strong resonances between the spin of the body and the mean motion of the ring particles, including higher-order resonances. Here we focus on the second order 2/4 and fourth order 2/6 resonances that are relevant for Chariklo and Haumea. The topologies of the phase portaits for those resonances are such that the 1/2 has a strong effect on ring particles, while the 2/6 resonances is much weaker.

1. Introduction

Both ringed-objects Chariklo and Haumea have shapes that significantly depart for spherical [2, 3]. As such they can create tesseral-type spin-orbit resonances involving the spin rate of the body, Ω, and the mean motion of the ring particles, n. More precisely, these resonances occur for

$$j\kappa = m(n - \Omega),\ m \text{ integer} \quad (1)$$

where κ is the particle epicyclic horizontal frequency, j is a positive integer and m is positive or negative integer (corresponding to inner and outer resonances, respectively). Since $\kappa \sim n$, the relation above reads

$$\frac{n}{\Omega} \sim \frac{m}{m-j}, \quad (2)$$

referred to as a $m/(m-j)$ spin-orbit resonance. From d'Alembert's rules, the corresponding resonance term of the perturbing potential is of degree j in the particle orbital eccentricity. The case $j = 1$ corresponds to Lindblad resonances (LRs), that have been extensively studied in the context of Saturn's rings, where they cause conspicuous spiral waves.

Higher-order resonances ($j > 1$) have received little attention because (i) they require higher expansions of the hydrodynamical equation of motion in the disk, a difficult task, and (ii) they cause the self-crossing of the streamlines, thus creating shocks and invalidating the usual hydrodynamical equations due to singularities in those equations.

2. Resonances around an elongated body

We examine in more details the cases $j = 2$ and $j = 4$, and discuss their impacts on rings.

Because they are small, Chariklo and Haumea's shapes depart significantly from spherical. Haumea is highly elongated, with an ellipsoidal shape of principal semi-axes $A \times B \times C = 1161 \times 852 \times 513$ km [2]. Defining the elongation parameter as $\epsilon = (A - B)/R$, where $R = \sqrt{3}(1/A^2 + 1/B^2 + 1/C^2)^{-1/2}$, we obtain $\epsilon = 0.43$ for Haumea, and a possible $\epsilon = 0.16$ for Chariklo [3]. The non-axisymmetric terms of the potentials are then proportional to $\epsilon^{|m/2|} \cos[m(\lambda - \Omega t)]$, where λ is the mean longitude.

For symmetry reasons, m is even, the strongest resonances occuring for $m = \pm 2$. Besides the first-order LRs ($j = 1$), the outermost and strongest resonance occurs for $m = -2, j = -2$, corresponding to $n/\Omega = 2/4$. Note that altough this resonance looks like the first order 1/2 LR, it is not because of it is second-order nature.

Another resonance of interest is $m = -2, j = -4$ (fourth-order), corresponding to $n/\Omega = 2/6$, as both Chariklo and Haumea's rings appear to orbit close to that resonance [2, 3].

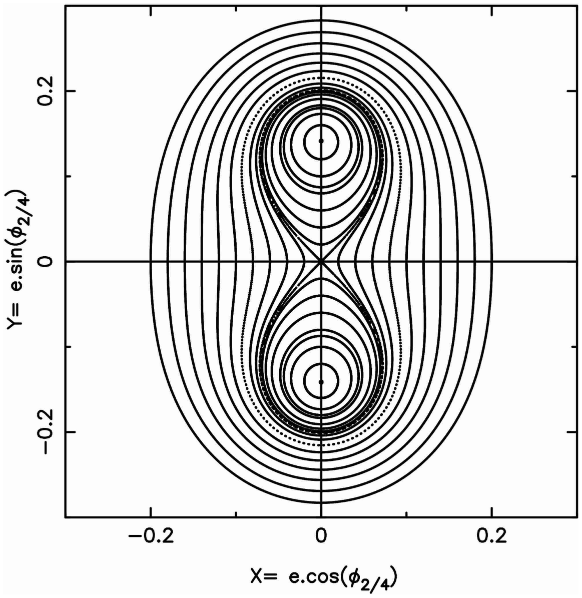

Figure 1: A typical phase portrait of the $n/\Omega = 2/4$ resonance near exact commensurability.

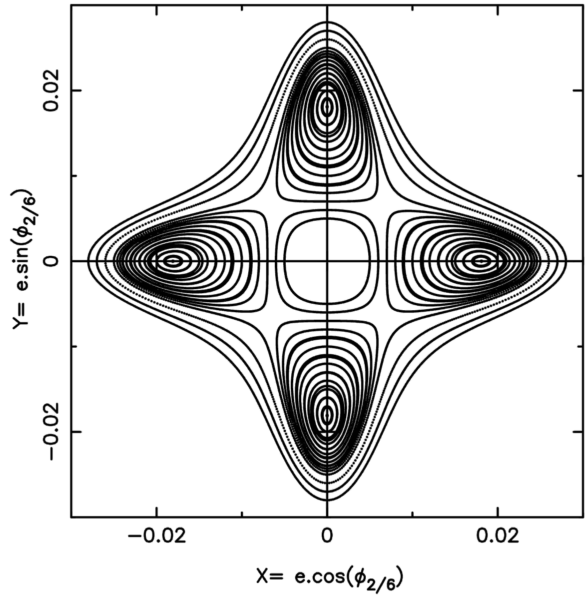

Figure 2: The same as Fig. 1 for the $n/\Omega = 2/6$ resonance.

3. Topology of the 2/4 and 2/6 resonances

Typical phase portraits of the 2/4 and 2/6 resonances are shown in Figs. 1 and 2. The relevant variables are $(X, Y) = [e \cdot \cos(\phi_{m/(m-j)}), e \cdot \sin(\phi_{m/(m-j)})]$, where the resonant angle is generically given by [4]:

$$\phi_{m/(m-j)} = \frac{m\Omega t - (m-j)\lambda - j\varpi}{j} \qquad (3)$$

A fundamental difference between the two resonances is that close to the exact resonance radius $a_{2/4}$ (more exacty in a neighborhood of width $\sim \epsilon \cdot a_{2/4}$), the 2/4 phase portrait has a hyperbolic, unstable point near the origin (Fig. 1), while near the 2/6 resonance, there is always a stable elliptic point (Fig. 2).

Collisions in dense rings tend to damp the orbital eccentricities of the particles, bringing them closer to the origin in the (X, Y) space. As a consequence, the 2/4 resonance forces the orbital eccentricity of the particles in the neighborhood $\sim \epsilon \cdot a_{2/4}$, while collisions damp the eccentricities near the 2/6 resonances, resulting in circular, mildly perturbed streamlines.

4. Summary and Conclusions

The second and fourth order spin-orbit resonances 2/4 and 2/6, respectively, between an elongated body and ring particles have structurally different phase portraits. This results in strong perturbations due to the 2/4 resonance, and much weaker perturbations from the 2/6 resonance. Applications to Chariklo and Haumea will be discussed.

Acknowledgements

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