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Interpolating light scattering properties using spiral curve on the sphere surface

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Abstract

We introduce a grid on a sphere constructed by a spiral curve. We describe the utilization of the spiral in interpolation and integration, together with cubic splines [1]. In particular, we show how the spiral is discretized on the sphere, and how the discretization is optimized. We apply the spiral interpolation and integration to specific scattering problems illustrating the power of the scheme.

1. Introduction

In Monte Carlo radiative transfer computations, the single-scattering phase function, or in a more general case, the Mueller matrix of the scatterer, is needed in the simulation. This property is needed when drawing new direction for the light ray after a scattering event. For complex particles this property needs to be computed beforehand for each particle type. Since the phase function or the Mueller matrix is a function of the scattered direction, some discretization is needed over the 4π solid angle for these directions and the associated Mueller matrices. In this work we present a scheme for discretizing the unit sphere, and to interpolate the scattering properties for directions between these discrete directions for which the scattering properties are computed and stored.

The discretization of the unit sphere for scattering computations have been studied before, for example in Okada et al. (2008) and in Penttilä et al. (2011) [2, 3]. However, these works have mainly concentrated on the numerical averaging for the random orientation properties, i.e., integration. With this study we concentrate on the interpolation of the scattering properties for an arbitrary scattering direction, using a set of discrete directions with known properties.

For the abovementioned interpolation task, we employ the spiral curve on the surface of a unit sphere. The discrete set of directions with known properties are sampled from this curve path. The discretization of the spiral curve is done so that the distances between the points on the curve is fixed.

2. Discretized spiral curve on the sphere

The spherical spiral path φ on the surface of a sphere is given below at (1),

$$\varphi(t) = (r\sin(t)\cos(kt), r\sin(t)\sin(kt), r\cos(t)) \tag{1}$$

where $t \in [0, \pi]$. The points of the discrete grid lie on the spiral path on the surface of the unit sphere (i.e., spherical spiral) [4, 5]. The spiral path is constructed with two parameters, the number of rounds around the sphere and the number of points on the path. Points are placed with constant arc length distances to each other along the spiral path.

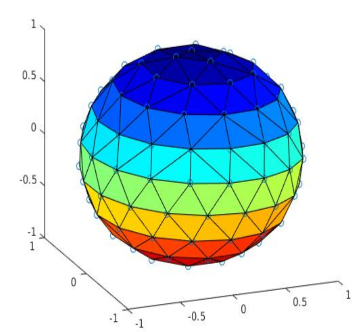

Figure 1: Spiral grid of a unit sphere

The number of points is optimized for the given number of rounds by an optimization algorithm. The discrete points can be connected into triangles by Delauny triangulation [6], and the algorithm finds the optimal number of points to so that these triangles are uniform in angles or edge lengths.

The interpolation for an arbitrary direction is carried out in the following way. At initialization, we form a cubic interpolation spline along the spherical spiral path with the abovementioned set of discrete points with known scattering properties. Then, we construct another longitudinal path from pole to pole so that it crosses the direction to be interpolated. This second path will cross the spiral path in several points, and the scattering properties can be interpolated at these crossing points with the spline on the spiral. Finally, a second cubic interpolation spline can be constructed on the longitudinal path with knots at the crossing points, with the scattering properties interpolated using the spiral spline at these knots. Using this second spline we can interpolate the properties for the given arbitrary direction.

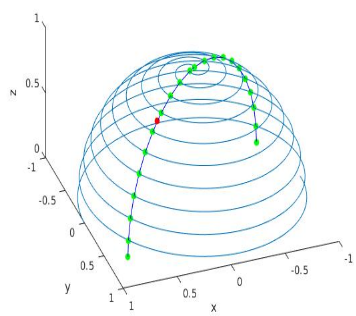

Figure 2: Paths of splines and crossing points on the hemisphere

Also integration over the unit sphere can be done with the weights to the grid points following the area of the corresponding Voronoi polygons [7, 8]. Voronoi diagram calculates fractions of the surface where the grid points are the closest inside the fraction.

Acknowledgements

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