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Behaviour of a stochastic parametric oscillator with application to dust particles originating in the inner Saturnian rings

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Abstract

The differential equation of a harmonic oscillator

$$\ddot{x} + \omega^2 x = 0 \tag{1}$$

naturally arises when linearising non-linear dynamical equations. Such oscillators easily become a stochastic problem, if the coefficient ω^2 depends on system parameters that are subject to random changes.
Assuming a parametric noise

$$\omega^2 = \omega_0^2(1 + \xi(t)) \tag{2}$$

we examine cases for both continuous and discrete stochastic processes $\xi(t)$ and find expressions for the Lyapunov Exponent

$$\lambda := \lim_{t\to\infty} \frac{1}{2t}\overline{\log E} \tag{3}$$

[1][2], with the energy-like expression

$$E(t) = \frac{1}{2}x^2(t) + \frac{1}{2}\dot{x}^2(t) \quad . \tag{4}$$

The Lyapunov exponent, which is the time-averaged exponential growth rate of $E(t)$, permits to distinguish between growing and decaying branches of the solution $x(t)$.

These results, together with general stability considerations are applied to charged dust particles, that are eroded in the inner rings of Saturn - for example as ejecta from impacts of interplanetary micrometeoroids. The charge q of those particles might change erratically due to varying charging fluxes or impacts/losses of single charge carriers. The first description is suitable for large particles that carry several additional charges, the second one for small particles, where single charging events have to be taken into account. These events are assumed to be poissonian distributed [3].

After an identification of the values of critical influences - i.e. the charging fluxes and the equilibrium charge, we compare the time scales of charging, stochastic growth (given by $1/\lambda$), and periodic motion of the epicyclically approximated particle motion.
This comparison finally allows to decide, for which particle sizes and initial positions in the rings, erratic motion due to charge fluctuations becomes important.

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