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The use of multifractal method for lunar topography analysis

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Abstract

The aim of the present work was to develop a fractal method for space topographic observations analysis. On the basis of the fractal method, modern lunar topographic models were compared. The lunar macrofigure models were built using harmonic analysis and expansion of altitude data from the dynamic selenocentric catalogue in a series of spherical functions. As a result, the values of fractal dimensions of the lunar relief anomalies were determined. The mean fractal dimension for the 4 lunar macrofigure models were produced. Analyzing the fractal similarity factors for various lunar surface models one may draw a conclusion on how similar these models are.

1. Introduction

The study of structure and evolution of celestial bodies involves various methods of statistical multiparametric analysis. Currently, one of the promising directions of studying heterogeneous natural objects' structure, materials, and properties is fractal geometry. For instance, the fractal analysis of the Solar system bodies' parameters has been conducted in the works [1, 2]. The fundamental property of fractal objects is similarity or scaling when zooming. The quantitative measure characterizing distribution of structure in space is fractal dimension D. Investigations of fractal dimensions allow studying not only the structure, but the connection between structure and its formation processes as well. Fractal structures have been found in the dynamic systems too [3]. The methods of fractal structure recognizing are used for heterogeneous surfaces' properties investigations and finding the similarity in certain parameters. In particular, the methods of fractal analysis allow for a quantitative description of the models of celestial bodies' surfaces.

2. The use of harmonic analysis for lunar topographic maps development

As a model describing the lunar relief we used expansion of altitude function in a series of spherical harmonics in the form of regression [3]:

$$h(\varphi,\lambda) = \sum_{n=0}^{N}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\cdot \bar{P}_{nm}(\cos\varphi) + \varepsilon\,, \quad (1)$$

where: φ, λ – latitude, longitude – known parameters of the lunar objects; C_{nm}, S_{nm} – normalized harmonic amplitudes; P_{nm} – normalized associated Legendre functions; ε – random error of regression.

Unfortunately, the series (1) is slowly convergent. For instance, to describe relief details varying through 1°, an order of expansion of about 180 is required, which causes the necessity of estimating $(180+1)^2$ coefficients (amplitudes) of expansion. Practically, dimension of the model (1) and therefore the order n should be specified on the basis of the number of objects almost evenly distributed over the sphere. Their number should from 5 to 15 times exceed the number of the objects estimated.

Within the regression modeling approach, we solved the overdetermined system (1) for various sources of hypsometric information. Along with the usual stages (postulating the model (1) and the amplitudes C_{nm}, S_{nm} estimation), the approach involves using a number of quality statistics including external measures, such as diagnostics of the basic LSM conditions observance. As LSM computational schemes, Gauss-Jordan and Householder ones were used.

3. Analysis of lunar topography using fractal geometry

Methods of determining fractal structures and properties of the objects are of great interest. Investigations of this kind allow for a quantitative description of the systems as follows: polymers, colloidal complexes, rough and porous surfaces, branching structures, plots of the Earth and planets, biological objects.

For each structure at the initial stage, according to the classic formula, the fractal dimension was defined which is sufficient to conduct a comparative analysis [4]:

$$D = \lim_{\delta \to 0} \frac{\ln N(\delta)}{\ln(1/\delta)}, \qquad (2)$$

where:

D – fractal dimension;

δ – the chords length between 2 profile points;

N – number of clusters.

The profile division into chords started with 24 in order to obtain an integer value of chords. It can be noted that the results are consistent with each other within the error limits. The matching of macrosurfaces for various lunar models on the basis of fractal analysis has not been done before in the international practice. In this work we are performing this procedure. 3 models of lunar surface were used. The first one was built with the harmonic method from ground-based observations. The second one was built using the “LRO” space mission data, while the third one – using “Kaguya” space mission. As a result, the mean values of fractal dimension were obtained: D (model 1) = 1,345, D (model 2) = 1,432, D (model 3) = 1,380.

4. Summary and Conclusions

In the present work, the lunar surface models were built on the basis of harmonic analysis and the dynamic selenocentric lunar objects coordinates data in order to develop models of lunar figure in the dynamic selenocentric coordinate system. The developed models were compared, which allowed for the estimation of the reliability of the relief represented in them. The mean values of fractal dimensions for 3 models of the lunar surface were determined. It was also found that a reference system of relief data did not influence the similarity estimations of the models containing symmetric information. Thus, it was revealed that using the fractal method for the relief similarity estimation allows for an investigation of reliability of the information represented in topographic systems.

Considering the fact that in the present work the models of the lunar surface displaying the same relief were taken, we may conclude the method of calculating fractal dimension allows obtaining reliable topographic similarity estimates. In addition, the values of fractal dimension indicate that the reference surfaces for altitude data do not influence the models’ similarity estimates.

The further use of the fractal comparative analysis at space measurements processing will certainly bring the results allowing to solve some problems of space astrometry. In particular, the fractal method may be used at physical surface analysis of Mars and Venus, asteroids, comet nucleus, gravitational fields of celestial bodies. These studies will be represented in the next works of the authors.

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References

[1] Turcotte, D.L.: A fractal interpretation of topography and geoid spectra on the earth, moon, Venus, and Mars, Journal of Geophysical Research, Vol. 92, Issue B4, pp. 597-601, 1987.

[2] Stepinski, T.F., Collier, M.L., McGovern, P.J. and Clifford, S.M.: Martian geomorphology from fractal analysis of drainage networks, Journal of Geophysical Research: Planets, Vol. 109, Issue 2, E02005 1 - E02005 12, 2004.

[3] Nefedyev, Y., Valeev, S., Mikeev R., Varaksina, N. and Andreev, A.: Analysis of data of “CLEMENTINE” and “KAGUYA” missions and “ULCN” and “KSC-1162” catalogues, Advanced in Space Research, 50, pp. 1564-1569, 2012.

[4] Peitgen, H.-O., Hartmut, J. and Saupe, D.: Chaos and Fractals, Springer Science + Business Media, Inc., pp. 1-894, 2004.