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Limit Cycles in a Toy Model of "Earth-like" Magnetotail Dynamics

Robert Burston
University of Bath, Bath, UK, rjb22@bath.ac.uk

Abstract

A toy model of "Earth-like" magnetotail dynamics capable of separately incorporating the Dungey and Vasyliunas Cycles is presented. This makes it applicable to Mercury, Earth, Jupiter and Saturn. The model is also capable of incorporating dynamical noise in the system. No previous model has this level of generality. Evidence is presented of the existence of limit cycles in the dynamics of the model. Limit cycles correspond to periodic behavior but the waveform need not be a simple sinusoid. The presence of limit cycles in the system dynamics implies that under some driving conditions, magnetotail plasmoid formation can be a predictably repeating process.

1. Introduction

The magnetospheres of the planets, Mercury, Earth, Jupiter and Saturn can all be described globally as having a compressed nose on the sunward side, up-stream in the solar wind and a long, stretched magnetotail on the night, down-stream side. When conditions are appropriate all four magnetotails can undergo a cyclical process of plasmoid formation driven either by the solar wind/Dungey Cycle (Mercury, Earth), atmospheric rotation/Vasyliunas Cycle (Jupiter) or both (Saturn) [1]. A toy model describing the dynamics of plasmoid formation, applicable to all of these planets is useful for understanding the variable responses to different driving conditions and how predictable the system is. Such a model is presented here. Although other types of behavior are possible (e.g. stochastic and chaotic modes) the results presented here focus on evidence for the existence of limit cycles (periodic behavior) at certain parameter values. The model is based on the analogy between magnetotail plasmoid formation and the formation of water drops by a leaking tap [2] and is an extension of the Shaw model of a leaky tap [3]. This models the leaky tap as a mass on a spring under gravity, with mass increasing at a constant rate. The analogy between the three systems is close because of the tension force involved; elasticity in the spring, water surface tension in the leaky tap and magnetic tension in the magnetotail.

2. The Model

The model is a form of non-linear relaxation oscillator given by the following equations:

$$F = mg - kx - C_{damp}\frac{dx}{dt} - C_{drag}\left(\frac{dx}{dt}\right)^2 \quad (1)$$

$$\frac{dm}{dt} = C_D \pm w_D + C_V \pm w_V \quad (2)$$

$$\Delta m = C_u \frac{dx}{dt}\bigg|_{x=x_c} \quad (3)$$

(1) represents the forces on the spring (from left to right, gravity, the elastic restoring force, damping by heat generation and viscous drag). F is the total force, m is the attached mass (not constant), k is the spring constant, x is the displacement, t is time and C_{damp} and C_{drag} are the damping and drag constants, respectively. (2) describes the continuous rate of change of mass and has four terms. C_D and C_V are the rates of increase of attached mass due to the Dungey and Vasyliunas Cycles, respectively. The time dependence of the values of C_D and C_V may be altered arbitrarily in the model. In this paper, however, each is set to be constant for the whole of any given model run. w_D and w_V are noise terms that modify the values of C_D and C_V. These may be set to zero (purely deterministic case) or to select from any pseudo-random distribution realisable in Matlab at each time step of the model run. These variations represent dynamical noise. (3) represents the instantaneous loss of mass when a plasmoid detaches at the critical

displacement, x_c. Δm is the instantaneous mass lost. C_u is a constant of proportionality that may take any positive finite value but is fixed for any given model run.

3. Results

For some sets of parameter values, the behavior of the model, as viewed in the system phase space, is that of an attracting limit cycle. A limit cycle is a closed loop in phase space. Once on a limit cycle, the system can never escape it. Limit cycles represent periodic behaviour but not necessarily a simple sinusoidal oscillation. An attracting limit cycle is a limit cycle that attracts nearby points in phase space on to it given sufficient time. Figure 1 shows a trajectory in the 3-D phase space of the model presented in section 2, above, that is an example of an attracting limit cycle. As time goes on, points on the trajectory get progressively lighter, as indicated by the colour bar. The trajectory can be seen to settle on a repeating distorted helix at late times. This is the limit cycle. Figure 2 shows the discrete masses lost over time for the same model run as represented in Figure 1. Time is on a log scale to clearly illustrate early behavior. It can be seen that the system rapidly settles into a pattern of releasing identical amounts of mass at constant time intervals. This is analogous to repeatedly forming plasmoids of uniform size at constant time intervals.

Figure 1: This is the example of an included figure.

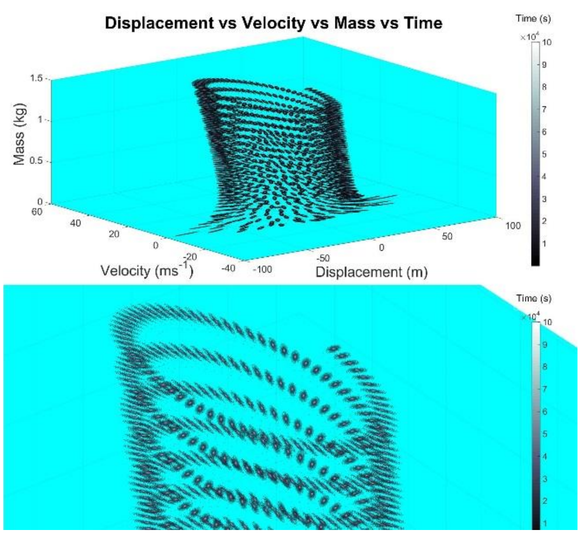

Figure 1: (Above) an attracting limit cycle in the model phase space. (Below) A zoom into the top region of the attracting limit cycle. It can be seen that at late times (lighter points) the trajectory has converged on to the centre of the approximately helical structure. This is the limit cycle.

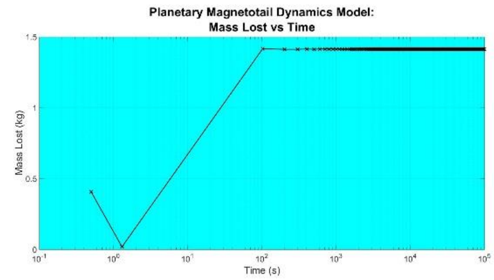

Figure 2: The discrete masses lost as a function of time (log scale). Note the rapid convergence to periodic behavior. The solid black line is an aid to the eye: masses are lost as discrete amounts at times marked by the black X symbol.

4. Conclusions

The model presented has attracting limit cycles in its phase space for some parameter value sets. These correspond to periodic behaviour. When viewed in terms of discrete mass lost, analogous to drops of water in the leaky tap case and plasmoids in the magnetotails case, the behaviour is seen as a brief transient followed by regular release of the same amount of mass at uniform time intervals. This is qualitatively the same as the behaviour of Earth's magnetotail at times of low to moderate substorm activity.

5. Acknowledgements

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