

## Photometric efficiency of a set of geometries

**F. Schmidt** (1), S. Bourguignon (2)

(1) GEOPS, Univ. Paris-Sud, CNRS, Université Paris-Saclay, Rue du Belvédère, Bât. 504-509, 91405 Orsay, France (2) Laboratoire des Sciences du Numérique de Nantes, École Centrale de Nantes, France (frederic.schmidt@@u-psud.fr)

### Abstract

The Hapke model [1] has been widely used to describe the photometric response of a surface (reflectance as a function of incidence and emergence direction). The angular sampling required to constrain the photometric parameters has been widely discussed, but often without strong mathematical basement. A recent study proposed to estimate the uncertainties using a Bayesian approach [2]. We introduce here the concept of “efficiency” of a set of geometries to sample the photometric behavior in the Bayesian framework. A set of angular sampling elements (noted a geometry) is efficient if the retrieved Hapke parameters are close to the expected ones. With numerical simulations, we compared different geometries and found that the principal plane with high incidence is the most efficient geometry among the tested ones. In particular, such geometries are better than poorly sampled full BRDF (Bi-directional Reflectance Distribution Function).

### 1. Introduction

The Bi-directional Reflectance Distribution Function (BRDF) is the core quantity to describe the photometric behavior [1]. It represents the same location pixel (for picture element), observed with various angular elements (angel, for angular element) [3]. Hapke proposed a semi-analytical model of the BRDF of a granular medium [1]. Many authors have been using it to analyze laboratory data [4, 5], telescopic observations [6], in situ data [7], remote sensing data [8], due to its relative simplicity and fast computation. Following our previous study [2], we do not discuss the realism of the photometric Hapke model, but focus on the data analysis point of view in order to determine the efficiency of a set of geometries to retrieve the Hapke parameters. We will consider here only single scattering albedo ( $\omega$ ), macroscopic roughness ( $\theta$ ), 2-parameter Henyey-Greenstein phase function ( $b$ ,  $c$ ). The opposition effects parameters ( $B_0$  and  $H$ ) are out of the scope of

this study since their dependence to small phase angle is obvious.

### 2. Method

In [2], a Monte-Carlo Markov Chain algorithm is proposed, which generates  $N_{\text{samp}}$  samples of the parameters of interest. Such samples are (asymptotically) distributed according to the *posterior probability density function*, which merges the likelihood of observed data and possible prior information on the parameter distribution.

In our experiments, we first fix the geometries (incidence and emergence directions). Then, we generate a synthetic observation set with the Hapke model using 12 different combinations (see Table 1). Then, we perform the Bayesian inversion [2] and track how close are the solutions in comparison to the known parameters.

	Configurations
$\omega$ (-)	0.1, 0.7
$\theta$ (°)	0.5, 25.0
$\{b ; c\}$ (-)	{0.1 ; 1.0}, {0.4 ; 0.4}, {0.8 ; 0.1}

Table 1: Photometric configurations of the Hapke parameters used in the estimation of the efficiency distance  $E$ . All 12 combinations of those 4 parameters are tested.

The efficiency distance  $E$  is simply computed by considering the proportion of samples that fall inside the correct interval among the  $N_{\text{samp}}$  samples that were drawn [9]. We then consider that a solution is good if it lies within a 1% error margin (1% smaller and 1% greater than the true value). For each of the 4 parameters, we compute the proportion  $I$  and the corresponding distance  $D = -\log(I)$ . The efficiency distance is the sum of all 4 distances for all 4 photometric parameters. Figure 1 shows the relationship between the efficiency distance  $E$  and the fraction of good retrieval.

In order to have statistically significant results, we computed the analysis 10 times with random

initialization. Each inversion has been computed with  $N_{\text{samp}}=100,000$  samples.

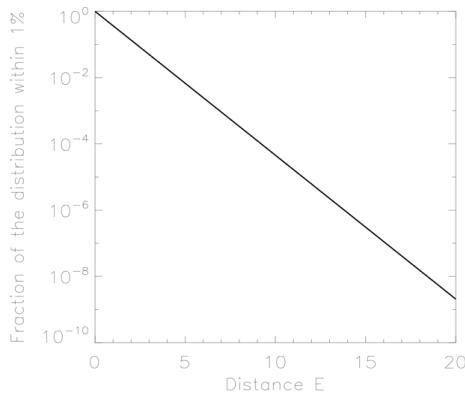


Figure 1: Fraction of the retrieval inside the 1% margin as a function of the efficiency distance E.

### 3. Results

Table 2 shows the results for several angular configuration. It seems that the 23 geometries proposed by [10] performed as well as a random configuration. Obviously, the worst configuration is the configuration that is perpendicular to the principal plane. Interestingly, the principal plane performs better than the full BRDF.

Table 2: Efficiency distance E for different angular configuration. 23 geometries proposed by [10], 23 geometries taken randomly, 23 geometries perpendicular to the principal plane, 23 geometries in the principal plane (with 75° incidence and up to 82° emergence), 64 BRDF with 40° and 60° incidence and 10-70° emergence.

Geometry	Global efficiency E
23souchon	11.37
23random	10.91
23worst	14.21
23pplane	<b>8.79</b>
64brdf	9.26

Figure 2 presents the effect of decimation (removal of intermediate geometries) from the best 23 principal plane configuration. The global efficiency distance E increases as the number of observation directions decreases and E rises significantly more for less than 5 angular configurations.

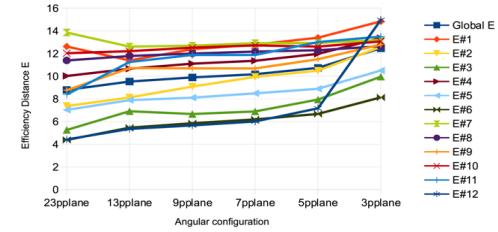


Figure 2: Efficiency distance E as a function of the number of geometry in the principal plane for 12 photometric behavior and the global E. The 23 geometries are decimated but extreme emergence (0° and 82°) are always kept

### 4. Conclusions

We introduced an index to measure the efficiency of a set of geometries (a collection of directions) to retrieve the proper Hapke parameters. Using the principal plane with high incidence angle seems the best solution with a limited number of directions. In particular, such geometries are better than poorly sampled full BRDF, even with a larger number of directions, due to the too large phase range. We also noticed that 5 directions is the minimum number of angular configurations in the best situation (principal plane) in order to expect well constrained photometric parameters [9].

### References

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