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Estimating coupled translational-rotational dynamics of solar system bodies

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Abstract

Next-generation planetary missions will provide an increase in the accuracy and diversity of measurements of solar system dynamics. This requires a reconsideration of natural body dynamical modelling strategies, to ensure that model errors are well below the measurement uncertainties.

Typically, rotation of solar system bodies is not estimated dynamically. Instead, free parameters in a kinematic model are fitted to available the data. For high-accuracy data, this requires the consideration of a large number of terms, complicating the estimation. We will present the mathematical framework, implementation and application of a method to propagate and estimate the coupled translational-rotational dynamics of a set of interacting bodies. Instead of libration parameters, estimated parameters include the initial rotational state and moment-of-inertia parameters.

We apply our methodology to Phobos, which exhibits a significant orbital-rotational coupling in its dynamics. We analyze the effect of our methodology on the estimation process, both for the currently available data, and tracking data from future lander missions.

1. Introduction

For the robust analysis of tracking data from planetary missions, the dynamics of solar system bodies under investigation should ideally be modelled to well below the observational accuracy and precision. For the analysis of data from future missions, dynamical models for natural bodies will need to be developed and implemented to beyond the current state-of-the-art of typical analysis software. One aspect that will need to be tackled is the manner in which rotational dynamics is estimated [Dirkx et al., 2014]. We present the methodology for a dynamical determination of coupled translational-rotational motion, as well as the implementation in the open-source Tudat software framework. We apply our methodology to the Martian moon Phobos, and discuss the impact that our approach can have on data analysis of current and future missions.

2. Coupled Dynamical Estimation

We define the state vector $\mathbf{x}$ of a single body as $\mathbf{x} = [\mathbf{r};\mathbf{v};\mathbf{q};\boldsymbol{\omega}]^T$, where $\mathbf{r}$ and $\mathbf{v}$ denote position and velocity, $\mathbf{q}$ the quaternion that transforms from body-fixed to inertial coordinates, and $\boldsymbol{\omega}$ the angular velocity vector in the body-fixed frame. To estimate coupled dynamics, the state transition matrix $\Phi(t,t_0) = \frac{\partial \mathbf{x}(t)}{\partial \mathbf{x}(t_0)}$ is required. To obtain this matrix, we derive the associated variational equations, which requires the calculation of:

$$\frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{x}} = \begin{pmatrix} \mathbf{0}_{3\times 3} & \cdot\mathbf{1}_{3\times 3} & \mathbf{0}_{3\times 4} & \mathbf{0}_{3\times 3} \\ \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}} & \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{v}} & \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{q}} & \frac{\partial \dot{\mathbf{v}}}{\partial \boldsymbol{\omega}} \\ \mathbf{0}_{4\times 3} & \mathbf{0}_{4\times 3} & \boldsymbol{\Omega}(\boldsymbol{\omega}) & \mathbf{Q}(\mathbf{q}) \\ \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \mathbf{r}} & \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \mathbf{v}} & \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \mathbf{q}} & \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \boldsymbol{\omega}} \end{pmatrix} \tag{1}$$

where $\mathbf{Q}$ and $\boldsymbol{\Omega}$ are linear in the entries of their input arguments. In the least-squares adjustment, we add the constraint for $||\mathbf{q}|| = 1$.

State estimation of spacecraft and natural bodies can provide a wealth of information on body interior properties, through the determination of a set of physical parameters $\mathbf{p}$ (gravity field coefficients, Love numbers, *etc.*), requiring a formulation for $\partial\mathbf{x}/\partial\mathbf{p}$. There is a direct link between a body's degree two gravity field coefficients, collectively denoted as $(C,S)_{2,0..2}$, and its inertia tensor $\mathbf{I}$. To provide a consistent estimation, as well as robust values for estimation uncertainties, the influence of $(C,S)_{2,0..2}$ on the inertia tensor is taken into account directly in the associated partial derivatives, through *e.g.* $\partial\mathbf{I}/\partial C_{20}$. The only 'new' parameters that are added to the estimated parameter vector (compared to the translational-only case) are then the initial rotational state $(\mathbf{q};\boldsymbol{\omega})$ and the body's mean moment inertia $\bar{I}$. In doing so, the influence of a body's interior structure is consistently mapped to both its rotational and translational dynamics.

The implementation of our methodology will be

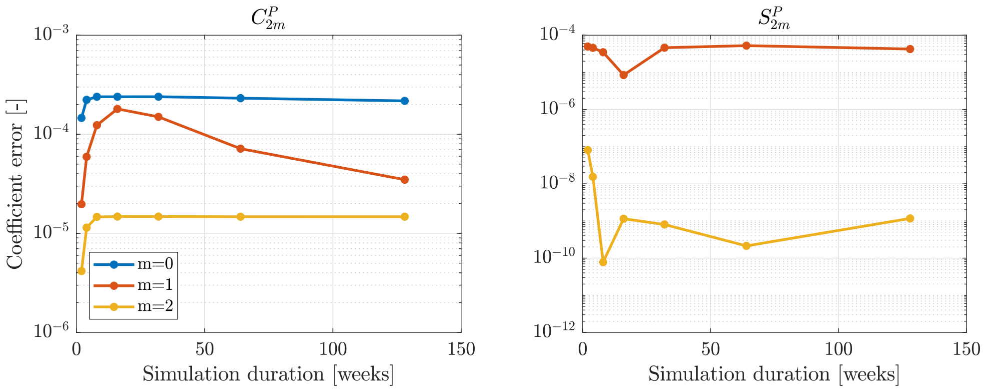

Figure 1: Estimation error of degree two gravity field coefficients of Phobos when omitting figure-figure effects during estimation.

freely available in the Tudat software [1]. This tool can be used for simulations of the coupled translational/rotational dynamics of any number of bodies, including the coupled estimation of arc-wise spacecraft orbit determination and single-arc translational/rotational dynamics of natural bodies.

3. Test Case - Phobos

There is a strong coupling between the translational and rotational dynamics of Phobos. This effect was exploited by [Jacobson and Lainey(2013)] to estimate Phobos' libration amplitude.

Future missions to Phobos will provide high-accuracy tracking data, requiring the analysis of dynamical effects that have thus far been considered negligible. Besides the coupled rotational-translational motion, the dynamical model will need to be extended to include the effect of figure-figure interactions. This is demonstrated in Fig. 1, which shows the effect that omitting figure-figure interactions during the estimation of Phobos' dynamics and gravity field will have on the estimation errors of its degree 2 gravity field coefficients.

[Le Maistre et al., 2013] and [Dirkx et al., 2014] used the rotation model of [Rambaux et al., 2012] to simulate the performance of the tracking of a lander to estimate, among other parameters, libration amplitudes. The decoupled rotational and translational dynamics was found to introduce errors in the dynamics for large perturbations of Phobos' state. Moreover, the estimation of an excessive number of libration parameters was required, leading to high correlations.

We will compare the estimation uncertainties using the classical approach (estimating one or more libration parameters), with our dynamical estimation. Both currently available data, and tracking data from future landers will be considered in our simulations. For future missions, we expect that our methodology will be especially advantageous, as it limits the number of estimated parameters, and reduces the potential correlations in the solution.

References

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[1] Code: `http://github.com/tudat` ; Documentation: `http://tudat.tudelft.nl`