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Benchmark for tidal deformation in planetary shells of variable thickness

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Abstract

Recent models of Enceladus's interior structure hint at large thickness variations of the ice shell. Realistic models of the tidally induced deformation and stress should reflect correctly these variations. Here we present results of our benchmarking efforts for recently developed approaches evaluating the tidal deformation of planetary shells with complex shape and rheology [1, 2, 3]. We assess the advantages and limitations of both approaches.

1. Introduction

Analysis of the libration, topography and gravity have shown that the Enceladus's ice shell is rather thin with large thickness variations e.g. [4, 5]. Crustal thinning at the south pole locally increases the stress and tidal dissipation [2, 3]. The predicted stress and heating enhancement could be partly responsible for the observed surface features and activity.

Studies of tidal deformation reflecting non-spherical shape of bodies are nevertheless rather rare in the planetary science community (see [3] for a summary). The standard approach is based on a spectral method requiring a spherically symmetric internal structure. Here we compare results of two approaches well suited to the computation of tidal deformations and dissipation in a shell of variable thickness.

2. Model

Here, we study the mechanical (quasi-static) response of a compressible viscoelastic shell of variable thickness on tidal loading. For describing the viscoelastic behavior, we assume the compressible Maxwell rheology characterized by the shear and bulk moduli and the viscosity η. The viscosity is generally nonuniform and we employ an Arrhenius-type [3] or a Frank-Kamenetskii-type temperature dependence for a conductive profile.

The tidal (loading) potential for a body on an eccentric synchronous orbit is described by

$$V = r^2\omega^2 e\left(-\frac{3}{2}P_{20}(\cos\vartheta)\cos\omega t + \frac{1}{4}P_{22}(\cos\vartheta)\left(3\cos\omega t\cos 2\varphi + 4\sin\omega t\sin 2\varphi\right)\right),$$

where t is the time, ω is the angular velocity and e is the eccentricity; P_{jm} are the associated Legendre functions for degree j and order m.

We use an analytical description of the ice shell thickness model

$$d = 23 - 12P_{20}(\cos\vartheta) + 4P_{30}(\cos\vartheta),$$

d in km [3], capturing well the main features of the gravity inversion models for Enceladus.

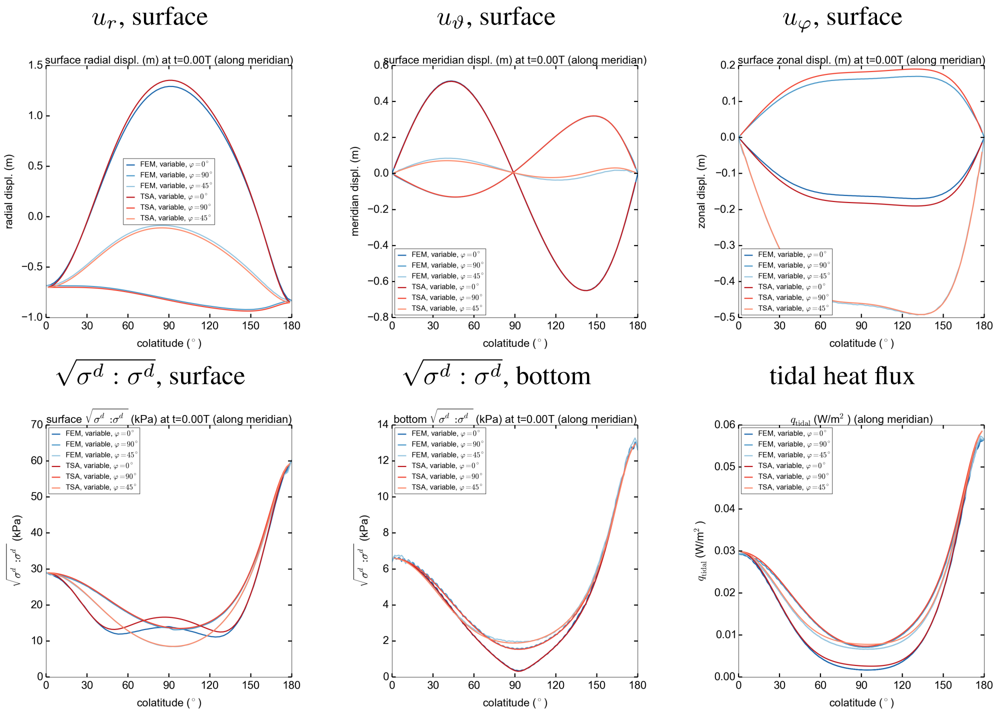
3. Methods

For evaluating the stress and displacement, we employ two approaches. The first method (finite element method or FEM, see [1, 2]) consists in integrating the Eulerian governing equation in the time domain. The equations are solved using three-dimensional finite element method and FEniCs package [6].

The second method (thin shell approximation or TSA, see [3]) takes advantage of the quasi-linear variation of the strain along the shell radius, which holds if the shell thickness is less than 10% of the surface radius. All variables are then integrated over the thickness leading to 2D equations. For a variable shell thickness, the tidal thin shell equations are solved as a system of coupled linear equations in a spherical harmonic basis.

4. Preliminary results

Preliminary results suggest that the two approaches agree well, with an error of the order of a few percents (see an example for low viscosity contrast in Fig. 1). The discrepancy for the tangential components of the displacement is mostly due to not fully converged solutions for the time domain method in regions with high viscosity (FEM). The difference in surface stress is slightly larger as the error of TSA could reach 15% at some locations on the equator [3]. The difference between TSA and FEM dissipation is partly due to the TSA error on stresses, and partly due to the bulk dissipation which is included in TSA. In general, TSA



u_r, surface

u_ϑ, surface

u_φ, surface

$\sqrt{\sigma^d : \sigma^d}$, surface

$\sqrt{\sigma^d : \sigma^d}$, bottom

tidal heat flux

Figure 1: An example of results (displacement $\mathbf{u}$, deviatoric stress σ and tidal heat flux) for variable ice shell thickness and small viscosity contrast (Frank-Kamenetskii-type dependence, $\eta_{\min} = 10^{13}$Pa.s, $\eta_{\max} = 10^{16}$Pa.s), TSA (red), FEM (blue), plots along meridians at the surface or at the bottom boundary

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is a fast and stable method allowing for large viscosity contrasts. On the other hand, TSA is intrinsically an approximate method: it is thus advisable to estimate the error for a spherically symmetric model before solving the full problem. In comparison, FEM is naturally significantly slower as it solves the full three dimensional problem, and it may suffer from numerical oscillation especially for the stress. On the other hand, fewer assumptions are employed and it can deal with broader applications, including faults [1].

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