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Effects of different equations of state on the interior structure of exoplanets

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Abstract

In this work we aim at quantifying the influence of different isothermal and thermally-dependent equations of state (EoS) on the computation of the interior of exoplanets. We perform an extensive parameter study to model the interior structure of a vast number of sub-Neptunian exoplanets of different composition, ranging from super-Earths consisting just of a metallic core and a silicate mantle, to sub-Neptunes including ice and gaseous layers. Each model run is performed with a number of different isothermal as well as thermally-dependent EoS. We find that for the rocky interior of an exoplanet, the choice of EoS has little influence on the characterization of the interior structure of the planet.

1. Introduction

One of the major aspects of current exoplanetary science is the characterization of the planetary interior.

A common approach to characterize the interior of a known exoplanet is the use of numerical models to compute an interior structure which complies with the measured mass and radius of the planet [1, 2]. In general, however, possible solutions are highly degenerate, with multiple, qualitatively different interior compositions that can match the observations equally well (Fig. 1). It is therefore necessary to run numerous interior structure models covering a wide range of possible interior parameters to constrain solutions. In order to not increase any intrinsic degeneracy, it is also important to look at the impact of different parametrizations for both the interior and atmosphere combined. In this work we will mostly focus on the interior while keeping the atmosphere very simple.

The majority of computation time for an interior structure is spent solving the equations of state (EoS). For large parameter studies it is therefore beneficial to use the simplest EoS which still provide an accurate characterization of the planet's interior. In this work, we aim to investigate the differences in model interiors the use of different EoS entails.

2. Numerical Model

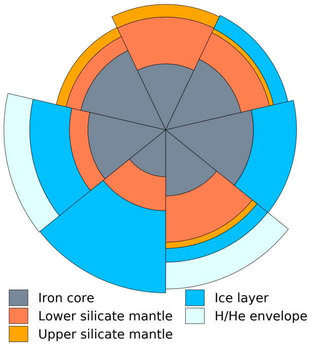

Figure 1: Seven interiors of an Earth-mass planet with varying core, ice and gas envelope mass fractions. The center top slice describes an Earth-like planet.

We employ a 1D structure model to construct sub-Neptunian exoplanets. A model planet consists of up to five layers: an iron-rich core, a lower silicate mantle composed of perovskite and magnesiowüstite, an upper silicate mantle composed of olivine and orthopyroxene, a potential water ice layer, and a gaseous envelope of varying composition (see Fig 1). The size of the core, the mantle as well as the ice layer are constrained by prescribed mass fractions. The interface between upper and lower mantle is defined by the plivine-perovskite phase transition at 23 GPa. We assume the interior to be convecting. The temperature then follows an adiabatic profile.

Each layer uses a prescribed equation of state to link pressure, temperature and density. We consider the following common formulations of EoS:

1. **3rd order Birch-Murnaghan [1, 3]:** This is a commonly used empirical EoS based on the finite

Eulerian strain. Temperature dependence is incorporated into the thermal expansion coefficient.

2. **Mie-Grüneisen-Debye [1, 3]:** In contrast to the Birch-Murnaghan EoS, the pressure is split into static pressure and thermal pressure terms. The thermal pressure is calculated based on the Debye model for specific heat. In the isothermal case, the Mie-Grüneisen-Debye EoS is equivalent to the isothermal Birch-Murnaghan EoS.
3. **Generalized Rydberg [4]:** This EoS is a generalization of the Vinet EoS [5], which is based on the interatomic repulsive potential. The thermal effect is incorporated as a pressure correction term, similar to the Mie-Grüneisen-Debye formulation.

The upper mantle uses the Birch-Murnaghan EoS. For the core, lower mantle and ice layer we utilize all EoS formulations described above. All models are tested with isothermal and thermally-dependent formulations.

For the moment, the gaseous envelope is modelled using a simple polytrope with polytropic index n=1.

We perform an extensive parameter study to assess the effect of different EoS on the observed radius of the modeled planet. Varied parameters are the planet mass, the mass fractions of each layer, the mantle potential temperature of the interior and the composition of the minerals constituting the material of the rocky layers.

3. Results

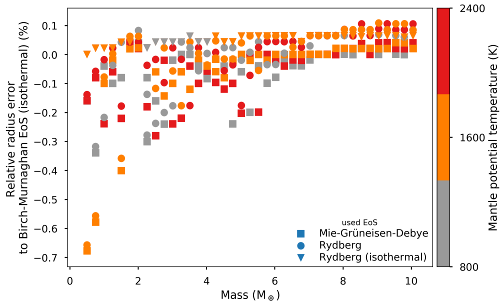

Figure 2: Relative radius errors of the EoS cases compared to the isothermal Birch-Murnaghan case for planet masses up to 10 $M_\oplus$ with a core-mass fraction of 0.3.

We find that in the case of an iron/silicate planet all EoS formulations result in computed planet radii which are less than 50 km apart in the most extreme case of a nearly molten planet. This amounts to a relative error of less than 0.7% in all cases studied (Fig. 2). This is an order of magnitude smaller than the precision we can expect for measurements of planetary radii in the near future (e.g. by the PLATO mission [6]). We conclude that for Earth-like planets the use of a simple isothermal EoS (in this case a 3rd order isothermal Birch-Murnaghan EoS) may be sufficient to accurately model the interior.

Acknowledgements

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