

# Coupling the internal and orbital evolution of close-in terrestrial exoplanets

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## Abstract

Exoplanets orbiting close to their host star are subject to extreme environments. Their internal dynamics are marked by pronounced tidal heating and high surface temperatures or temperature contrasts. Their orbital evolution, as well as the evolution of the spin rate, is greatly influenced by the tidal interaction with the host star, which could lead the planet into one of the stable spin states (either a spin-orbit resonance or a pseudosynchronous rotation) and – on longer time-scales – even circularize its orbit. The rate and the exact form of such a process depends on the rheological parameters of the planet, which, in turn, are linked to the thermal evolution via their temperature dependencies.

Here, we introduce a semi-analytical model of coupled thermal-orbital evolution of a single terrestrial exoplanet orbiting a single star and present some of its applications on several currently known planetary systems.

## 1. Model and Methods

In order to describe the long-term evolution of a rocky planet without atmosphere, we combine a tidal evolution model for viscoelastic bodies with parametrized (1D) mantle convection.

The calculation of the orbital evolution is based on the Darwin-Kaula expansion of the tidal potential and Lagrange's planetary equations [3], which, together with the conservation of the total angular momentum, read

$$\frac{da}{dt} = - \sqrt{\frac{a}{\mathcal{G}(M+m)}} \sum_{lmpq} k_l \sin \varepsilon_{lmpq} \frac{2r_p^{2l+1}}{a^{2l+2}} \times B_{lm} G_{lpq}^2 F_{lmp}^2 (l-2p+q), \quad (1)$$

$$\frac{de}{dt} = - \sqrt{\frac{1-e^2}{a\mathcal{G}(M+m)}} \sum_{lmpq} k_l \sin \varepsilon_{lmpq} \frac{r_p^{2l+1}}{ea^{2l+2}} \times B_{lm} G_{lpq}^2 F_{lmp}^2 [\sqrt{1-e^2}(l-2p+q) - (l-2p)], \quad (2)$$

$$\frac{d(C\Omega)}{dt} = - \frac{1}{2} \sqrt{\frac{\mathcal{G}M^2(M+m)}{a}} \sqrt{1-e^2} \frac{da}{dt} + \sqrt{a\mathcal{G}M^2(M+m)} \frac{e}{\sqrt{1-e^2}} \frac{de}{dt}. \quad (3)$$

Here,  $a$  is the semi-major axis,  $e$  is the eccentricity,  $\Omega$  symbolizes the rotational frequency,  $\mathcal{G}$  stands for the gravitational constant,  $m$  and  $M$  are masses of the two bodies,  $G_{lpq}$  and  $F_{lmp}$  are the eccentricity and the inclination functions, respectively,  $r_p$  is the planetary radius and  $B_{lm} = \mathcal{G}M \frac{(l-m)!}{(l+m)!} (2 - \delta_{0m})$ . For the sake of simplicity, we neglect the tides raised on the star due to the planet, set the inclination and the axial tilt of the planet to zero and consider the moment of inertia about its rotation axis  $C$  constant. The eccentricity functions  $G_{lpq}$  are computed as the Hansen's coefficients  $X_{(l-2p+q)}^{-(l+1), (l-2p)}$  either numerically, from their definition, or using recurrent formulae [1].

The internal structure of the planet and its rheological properties are represented by the frequency dependent tidal Love numbers  $k_l(\omega_{lmpq})$ , describing the change in the external potential due to the planet's tidal deformation, and by the phase lag between the tidal and the disturbing potential  $\varepsilon_{lmpq}(\omega_{lmpq})$ . We calculate these parameters adopting the method of [5] for a tidally loaded body and considering a layered spherical model planet with solid inner core, liquid outer core and a viscoelastic mantle, whose upper part forms a highly viscous lithosphere.

Together with the evolution of the orbit and the spin rate, we evaluate the average tidal heating and update the internal structure of the planet and the viscosities

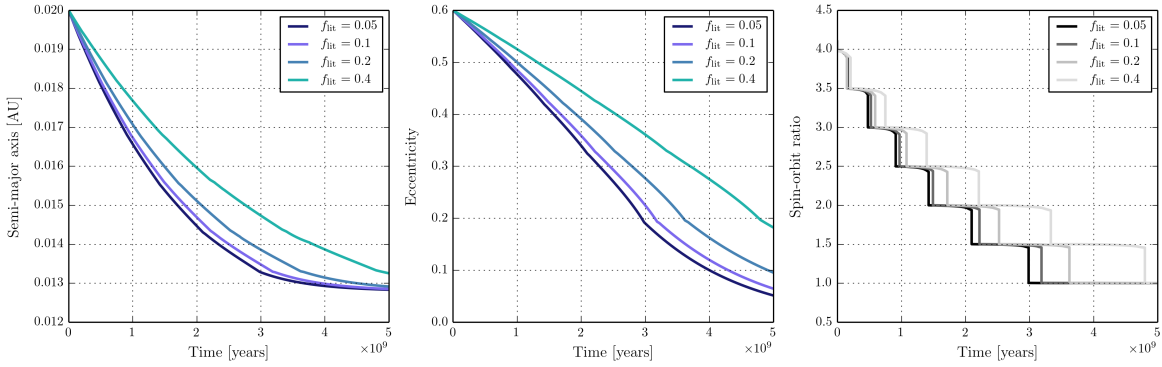


Figure 1: Long-term evolution of the semi-major axis, the eccentricity and the spin rate, computed for an Earth-size planet orbiting a red dwarf ( $M = 0.1 M_{\text{Solar}}$ ). Effect of the thermal lithosphere thickness.

of its layers according to the changes in the thermal structure. Subsolidus convection in the stagnant lid regime is computed using a 1D parametrized model of a mantle heated both volumetrically (due to the tidal dissipation and the decay of radiogenic elements) and from below (e.g., [2]). The viscosity  $\eta$  of the mantle follows an exponential temperature dependence of the form

$$\eta = \eta_0 \exp\left(\frac{E^*}{R} \frac{(T_{\text{ref}} - T)}{T_{\text{ref}} T}\right) \quad (4)$$

with  $\eta_0 = \eta(T_{\text{ref}})$ , where  $T_{\text{ref}} = 1600$  K is the reference temperature,  $E^*$  is the activation energy for viscous deformation and  $R$  is the universal gas constant.

## 2. Preliminary results

For the static model, without mantle convection, we perform several parametric studies concerning the dependence of the Love numbers and the tidal torque on the internal structure and continue with assessing its effect on the orbital evolution. Figure 1 depicts orbital and spin-rate evolution of an Earth-size planet, depending on the ratio  $f_{\text{lit}}$  between the thickness of its thermal lithosphere and the thickness of the mantle. Average viscosity of the mantle is  $10^{16}$  Pa.s; viscosity of the lithosphere increases logarithmically from its base to the surface, where it reaches its maximum value of  $10^{24}$  Pa.s. Varying thickness of the lithosphere affects the rate of the orbital eccentricity decay and, as a result, determines the stable spin-orbit resonance.

The results of these parametric studies will be com-

pared to the time-evolving, convecting model with tidal heating and applied on several currently known rocky exoplanets. We are also planning to connect the semi-analytical model with numerical computation of Love numbers [4], which would enable us to study moons or planets with heterogeneous mantle viscosity structure.

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## References

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