



The equilibrium points of spinning-top shape asteroids

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Introduction

Asteroids are reminiscent bodies of the Solar System that are irregularly shaped, though some of them are nearly spheroidal. Despite of their irregular shape, some asteroids have similar characteristics. The spinning-top asteroids are characterized as oblate shapes with a pronounced equatorial bulge, as well as their fast rotation.

To understand the dynamics around these asteroids, we calculate the equilibrium points [1] of some spinning-top bodies, considering their irregular shape. The equilibrium points are points where there is a balance between the gravitational and centrifugal forces. Thus, when the body is spinning faster, the equilibrium points may be closer to the body, and some of them could touch the body's surface or even be inside the body. Besides the location of the equilibrium points, we also examined their topological structure according to the eigenvalues of the characteristic equation in the complex plane.

Location and classification of the equilibrium points

We considered a set of spinning-top asteroids to compute their respective equilibrium points: the Alpha bodies of the triples systems 2001 SN₂₆₃ [2] and 1994 CC [3], the Alpha body of the binary system 1999 KW4 [4], the asteroids 2008 EV5 [5,6], Ryugu [7] and the retrograde model of the asteroid 1950 DA [8,9].

In this study, we identified that the Alpha body from the triple system 2001 SN₂₆₃ has the larger number of equilibrium points, with 13 points. Among these points, the E_{13} point is the central one, the point E_5 is an inner point and the remaining are close to the surface of the body, as if they were contouring the surface (Fig. 1).

The Alpha bodies from the systems 1994 CC and 1999 KW4, the asteroids 2008 EV5, and Ryugu have seven equilibrium points. In the same way as the equilibrium points of the 2001 SN₂₆₃ Alpha, the equilibrium points of the Alpha bodies from the systems 1994 CC and 1999 KW4 are close to or inner to their surfaces. For the Alpha 1994 CC, only one equilibrium point is outside of the body.

The asteroids Ryugu and 2008 EV5 also have points close to the surfaces. However, these points are not so close as the other bodies analyzed in this study. Since the rotation period of Ryugu is slower (7.63262 hours) it is expected that its points are not so close to the surface.

For the asteroid 1950 DA we calculated the equilibrium points for different bulk densities (3.5 g cm⁻³ to 1.0 g cm⁻³). We find nine equilibrium points for the larger bulk density and in this case all

points are close to the surface. However, as we decrease the bulk density, the equilibrium points move closer to the body's surface, even going inside the body for the smaller densities. Consequently, the number of points decreases, reaching the case of having only a single equilibrium point. In this case, the cohesive forces are important to keep the body's shape [9].

In terms of topological classification, we verified some similarities among all the points we calculated. All the odd points are classified as Saddle-Centre-Centre, while the even points are divided into two classifications: Sink-Source-Centre and Centre-Centre-Centre, being the Centre-Centre-Centre the only linearly stable points.

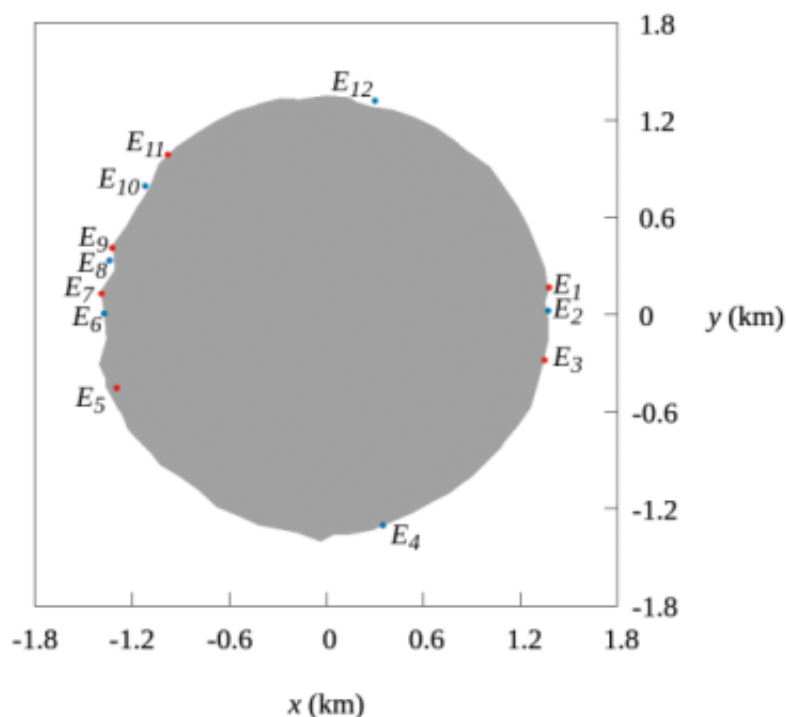


Figure 1: Location of equilibrium points of the body Alpha from the triple system 2001 SN₂₆₃ in the XY plane.

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