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Equivalence between elastic, viscoelastic, and viscous isostasy

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1. Introduction

Isostasy explains why observed gravity anomalies are generally much weaker than what is expected from topography alone, and why planetary crusts can support high topography without breaking up [1]. The apparent simplicity of the concept – buoyant support of mountains by iceberg-like roots – is belied by the debate that has been going on for over a century about ‘equal mass’ and ‘equal pressure’ prescriptions [2]. Since these isostatic models only differ at the planetary scale, it has not caused a problem for the application of isostasy to Earth because of its division in tectonic plates (large-scale geoid anomalies due to mantle convection are another reason). By contrast, isostasy on icy moons and dwarf planets is immediately faced with the problem of defining correctly isostasy at the largest scales. For this purpose, new isostatic models based on the minimization of stress [3], on time-dependent viscous evolution [4], and on stationary viscous flow [5] have recently been published (these models have historical precedents not cited here). The multiplicity of isostatic approaches is too much of a good thing. I will show that these new isostatic approaches are mostly equivalent.

2. Isostasy with Love

Isostasy can be formulated very generally as a loading problem using the ‘Isostasy with Love’ approach sketched in [3]. In Airy isostasy, the shell is loaded by a surface load (‘L’) and a bottom load (‘I’ for internal). The bottom-to-surface loading ratio ζ_n depends on the harmonic degree n and must be determined with the chosen isostatic prescription.

In elastic isostasy (‘E ’), the bottom-to-surface shape ratio can be expressed in terms of radial Love numbers at the surface or bottom of the shell:

$$S_n^E = \frac{\hat{h}^{\rm L}_{\rm bott} + \zeta_n^E \, \hat{h}^{\rm I}_{\rm bott}}{\hat{h}^{\rm L}_{\rm surf} + \zeta_n^E \, \hat{h}^{\rm I}_{\rm surf}} \, ,$$

where the ‘hat’ denotes deviatoric Love numbers, defined by subtracting the fluid value from Love numbers.

In viscoelastic isostasy (‘V ’), the shape ratio is given by a very similar expression, except that the Love numbers (and possibly the load too) are now time-dependent:

$$S_n^V = \lim_{t\to\infty} \frac{\hat{h}^{\rm L}_{\rm bott}(t) + \zeta_n^V \, \hat{h}^{\rm I}_{\rm bott}(t)}{\hat{h}^{\rm L}_{\rm surf}(t) + \zeta_n^V \, \hat{h}^{\rm I}_{\rm surf}(t)} \, .$$

For elastic isostasy, I consider here the model of Zero-Deflection Isostasy (ZDI), which is closely related to Minimum Stress Isostasy (MSI) but is simpler to formulate. In ZDI, the isostatic prescription states that the radial deflection vanishes at the surface (it could alternatively be zero at the bottom of the shell, but let us keep it simple). If the shell is homogeneous and the core is rigid, analytical formulas can be generated with the elastic propagator matrix method. For example, the topographic ratio (closely related to the shape ratio but taking the geoid into account [5]) is given by

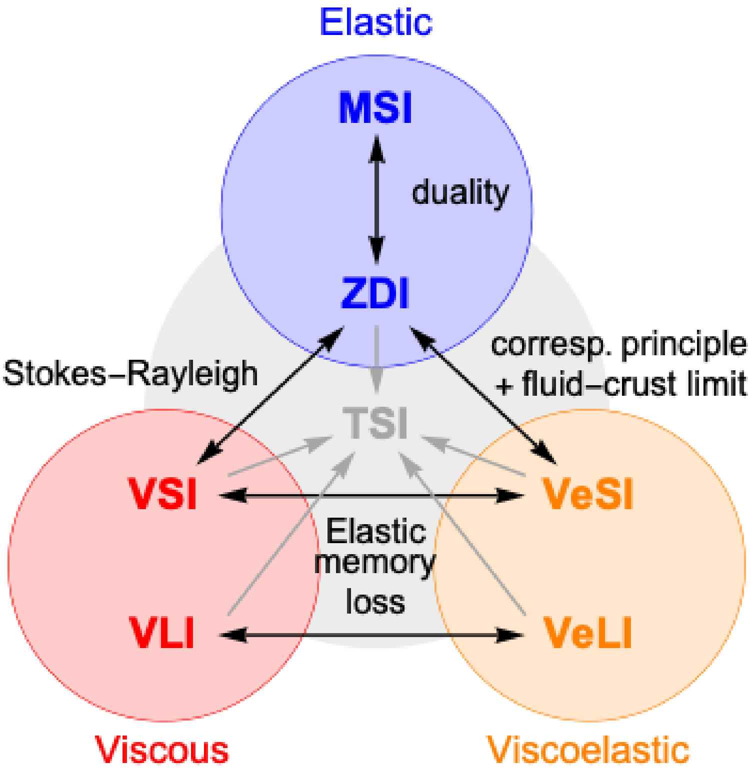
$$T_n^{\rm ZDI} = -\frac{\rho}{\Delta\rho} \, \frac{g_{\rm surf}}{g_{\rm bott}} \, f_n(r_{\rm bott}/r_{\rm surf}) \, ,$$

where $f_n(x)$ is a degree-dependent function which is known in analytical form.

In viscoelastic isostasy, similar analytical formulas can be obtained, using the correspondence principle, for Visco-elastic Isostasy with constant Load (VeLI) and with constant surface Shape (VeSI). In the viscous approach, analytical formulas for Viscous Isostasy with constant Load (VLI) or constant Shape (VSI) can be obtained using the viscous propagator matrix approach.

3. Visco-Elastic equivalences

The comparison of analytical formulas for the shape ratio, topographic ratio, or compensation factor in elastic/viscoelastic/viscous isostasy demonstrates the equivalences shown in Figure 1. Isostatic models thus belong to two independent groups: the elastic/stationary approaches ZDI-VeSI-VSI (plus MSI, not discussed here), and the time-dependent approaches VeLI-VLI. In the thin shell limit, the two groups yield the same predictions as Thin Shell Isostasy (TSI) which was initially obtained through stress minimization in a thin shell [6]. At low harmonic degree, TSI is a good approximation for Europa but a bad one for Enceladus. At high harmonic degree, shell thickness always matters and the various models give slightly different predictions of geoid anomalies.



These equivalences hold beyond the simple analytical model based on a homogenous crust and a rigid core. The equivalence between ZDI and VSI can be understood in terms of the Stokes-Rayleigh analogy relating elastic and viscous solutions with analogous boundary conditions [7]. This equivalence extends to viscous models with depth-dependent viscosity [5], which correspond to elastic models with depth-dependent shear modulus. The equivalence between ZDI and VeSI can be proven using the correspondence principle and the invariance of elastic isostasy under a global rescaling of the shear modulus of the crust (the latter implies that ZDI can be computed in the fluid-crust limit). The equivalence between VSI and VeSI, on the one hand, and between VLI and VeLI, on the other, can be understood as due to memory loss of the initial elastic response.

In conclusion, the elastic, viscoelastic, and viscous approaches to isostasy are not that different. The focus should now be on choosing the right boundary conditions, which express different physical pictures. In particular, viscous/viscoelastic isostasy with a constant load represents a completely different loading history from viscous/viscoelastic isostasy with a constant surface shape maintained by a time-dependent load at the bottom of the shell.

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