# **Correcting the Atmospheric Refraction of Fireball Observations at Low Elevation Angles and Significance of the Correction**

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# Abstract

We introduce a new atmospheric refraction correction method which allows to retrieve fireball position with high accuracy without the need to consider at which distance from the observer or height above the Earth's surface the fireball is situated. Traditional refraction correction is valid for objects positioned at infinite distance and it overcompensates when an object is situated inside the atmosphere. In this numerical study the overcompensated correction is reduced by artificially increasing the observing site height above the sea level, called the delta-z correction. We use analytically derived formula for the delta-z correction with different refraction models and compare these results to the numerical solution where light ray is traced through the atmosphere. Ray tracing technique is implemented on the finely meshed atmospheric layers in order to derive value of the correction. We parametrize the viewing angle and the fireball height above sea level in order to define whether this delta-z correction is significant or negligible. Significance is defined by studying the errors caused by the observed horizontal altitude, height of the fireball above the sea-level, and height of the observing site. We find that the delta-z correction should be performed if a fireball is observed within 20 degrees altitude above the horizon or with negative altitudes. We also find that delta-z correction is always accurate if fireball is situated 20km or higher above the sea level and hence it can be safely applied in processing of all observational cases of fireballs.

#### Introduction

Accurate directions are very essential in deriving fireball entry track and velocity. Errors in this affect calculated solar system orbit, dark flight simulations and calculated strewn field area. It is essential to consider atmospheric refraction.

When the fireball is apparently at low elevation angle, the full correction of the refraction makes the correction too big. A considerably large effect is caused by the tray curvature between the fireball and the observing station. If the elevation angle is corrected by the full refraction, this gives the fireball position significantly below its actual position. Even though the need to lift observing site artificially has been understood for years (Hansen, 1838 and Wittmann, 1997) it was considered to be useful mainly for satellite observations (Green 1985). Nowadays it is not sufficient for ex. satellite geodesy (Seeber, 2003).



**Figure 1.** A light-ray from an entering meteor is coming from a point A. An observer sees the meteor at an apparent angle H from a point B due to the atmospheric refraction R. Lines from the point A and C are parallel. In case refraction did not occur the original light-ray would propagate above the observing site at height  $\delta z$ . Correcting only refraction would result in faulty starting point C.

Figure 1 illustrates the case when we are observing entering fireball. The true position of the object is driffted in vertical direction of observing site if we are using the full refraction correction. We define this drift as  $\delta z$ . We elevate the observing site artificially by amount  $\delta z$ . The direction of the object with a full refraction correction then converges with the true position of the object. This  $\delta z$  value can be derived into analytical formula for spherically symmetric atmosphere (Green, 1985)

$$\delta Z = r_0 (n_0 \sin(90 - H) / \sin(90 - H + R) - 1)$$
(1)

where H is apparent angle, R is full refraction for H,  $r_0$  is radius of the lowest air shell and  $n_0$  is the corresponding refractive index of air. So far we have not encountered any cases where  $\delta z$  values were corrected using Green model. Green model can be used with any refraction model. One generally used model is Bennett's model (Bennett, 1982).

$$R = (0.28*P / T + 273.15) / (tan(H + 7.31/(H + 4.4))) / 60$$
<sup>(2)</sup>

where T and P are the temperature in degrees Celsius and the pressure in millibars at the observing site. Unfortunately this Green model is highly dependent of the refraction model and is therefore very delicate for errors in refraction values, as shown in the last column of Table 1. This Bennet's refraction model used in equation 1 returns overcompensated  $\delta z$  corrections.

Furthermore, Green model returns  $\delta z$  value for light ray penetrated through whole atmosphere. However, a bolide (or any other object) can be located in the atmosphere and therefore refraction is not applicable in this manner. The full refraction correction would be overcompensating the position of the bolide if used independently or in the Green model.

## **Method: Ray Tracing**

We generated data numerically using ray tracing method. Atmosphere was divided into 10 meter thick co-centric spherical shells from the mean sea level (i.e. at height = 0m) to upper bound (86000m). Higher atmospheric layers were neglected due to insignificant contribution to cumulative refraction. Air pressure, temperature and density for each layer were adapted from ISA-model (U.S. 1976). Refraction occurred at each layer boundary according to the **Snell's law of refraction**. Refractive index n<sub>0</sub> at the sea level was calculated using a complex Ciddor Equation (Ciddor, 1996) with values: vacuum wavelength = 570 nm, CO<sub>2</sub> content = 0 ppm, temperature = 15 °C, atmospheric pressure = 1013,25 mbar and air humidity = 80%. Hence we gained a value for refractive constant  $n_{air} = 0.000276567$ . Refractive indices for layers were calculated using equation

$$n_i = 1 + n_{air} * (P_i / P_0) * (T_0 / T_i)$$
(3)

Hence we gain refractive index for sea level  $n_0 = 1.000276567$ . Ray tracing was performed starting from the observer. Intergrating this way the actual position of the light ray can be tracked and geometric parameters defined.

In Table 1 we list total refractions and gained  $\delta z$  values. We compare Bennets refraction model with ray tracing method. Both are used as an input to Green model of  $\delta z$ .

Light ray can be observed with negative elevation angles, below the theoretical horizon, if the observer is at the higher site. This is illustrated in Figure 2. On the other hand, light ray passes through fewer atmospheric layers if the observed elevation angle is positive. We list  $\delta z$  values for observing site located at 1000 m.a.s.l. in Table 2.

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		Refraction	S	δz values			
Apparent angle [°]	Total refraction as RT model [°]	Total refraction as Bennett's model [°]	Difference in refraction RT - Bennett's[°]	ðz [m] as RT model	δz [m] as Green model, refraction as RT	oz [m] as Green model, refraction as Bennett's	
0	0.5334	0.5658	-0.0324	2040	2040	2075	
1	0.3906	0.3992	-0.0087	1153	1153	1143	
2	0.2951	0.2989	-0.0038	701	701	688	
3	0.2332	0.2354	-0.0022	456	456	444	
4	0.1907	0.1926	-0.0019	315	315	300	
5	0.1604	0.1622	-0.0018	227	227	209	
6	0.1378	0.1396	-0.0019	170	170	148	
8	0.1068	0.1086	-0.0019	105	105	75	
10	0.0866	0.0885	-0.0018	71	71	34	
12	0.0726	0.0744	-0.0017	51	51	9	
14	0.0623	0.0639	-0.0016	38	38	-7	
16	0.0544	0.0559	-0.0015	30	30	-18	
18	0.0482	0.0495	-0.0014	24	24	-27	
20	0.0431	0.0444	-0.0013	20	20	-32	

RT refers to ray tracing method. In the first section of the table we compare refractions at sea level calculated using RT and Bennet's equation (2). In the second section we compare calculated  $\delta z$  values.

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Apparent angle [°]	Total refraction RT-model [°]	δz [m] RT model	δz diff. [m] (sea level - 1000m)
-0.9	0.7049	3318	-
-0.8	0.6756	3095	
-0.7	0.6490	2894	
-0.6	0.6296	2721	
-0.5	0.5980	2530	
-0.4	0.5748	2369	
-0.3	0.5463	2210	
-0.2	0.5383	2092	
-0.1	0.5081	1951	
0	0.4867	1831	209
1	0.3560	1032	121
2	0.2687	626	75
3	0.2121	407	50
4	0.1734	280	35

 $\delta z$ -values for observing site situated 1000 m.a.s.l. are shown in the third column. The second column shows the full refraction (for star) observed at corresponding apparent angle. Differences to the  $\delta z$ -values for sea level are shown in the fourth column. Atmosphere was divided into 2 meters thick layers for negative angles. Notice dashed line for separation. Values are rounded four decimal places.

## **Error estimation**

In this study we considered only geometric error. Atmospheric anomalies were not examined. The height of the fireball from the ground level affects the error of the full refraction correction. Fireball at the lower height from the sea level requires smaller correction to  $\delta z$  value. This difference, error in  $\delta z$ , is illustrated in Fig 3. Ray tracing method was applied to objects at the lower atmospheric layers in order to study the error of the  $\delta z$  correction. We studied  $\delta z$  errors for objects situated at 10000, 15000, 20000, 25000 and 30000 m.a.s.l. (Table 3).

Table 3.						
Apparent		Object's height above sea level [m]				
angle [°]	Full <b>δ</b> z	10000	15000	20000	25000	30000
0	2040.429	124.235	42.395	15.374	5.853	2.331
1	1153.166	116.242	40.389	14.798	5.672	2.270
2	701.180	97.727	35.410	13.313	5.193	2.104
3	456.437	77.624	29.447	11.421	4.556	1.877
4	314.514	60.553	23.894	9.542	3.892	1.632
5	227.109	47.370	19.275	7.888	3.282	1.399

 $\delta z$  errors for different apparent angles when the object is located in the lower atmosphere (height  $\leq$  30000 meters).  $\delta z$  errors are shown for five different heights. The error is the vertical difference of the true location of the object compared to the full  $\delta z$ . This is illustrated in Fig 3.



**Figure 3.** A meteor is situated at point D with height h. Light ray follows the same path as it would be coming from point A.  $\delta z$  correction has an error in this case.

## **Results & Discussion**

- Applying the ray tracing method for  $\delta z$  calculations is arduous procedure and the presented simplifications are shown to be suitable for sea level observations (Table 1). This can also be included into the analysis software to be taken into account automatically.
- This simplification is appropriate if the purpose is to solve the 3-dimension luminous flight track for sea level observation.
- Green model is sufficient to be used for raising artificially observing site height. This  $\delta z$  correction can be used for actual meteor.
- Meteor height does not need to be considered for typical meteorite candidate (Sansom et al., 2019) as shown in Table 3.
- Green model does not involve information about the atmospheric condition and hence it is delicate for refraction model variations (Table 1).
- For higher observing stations (>1000m)  $\delta z$  values differ significantly in reference to sea level.
- Green model might be used for higher observing stations and negative apparent altitudes but defining the corresponding refractive index and refraction values is troublesome.

In this study we assumed that the actual air characteristics (such as temperature at the observing station's location) are not needed to be taken into account. This simplification, however, does not consider e.g. atmospheric inversion that can largely affect the results (Lyytinen et al. 2016), but is not easy to implement. This would require the application of the described ray tracing technique using the actual atmospheric data between the fireball and all the observation stations.

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