

Estimation of the interior density of a small body given its gravity field

Abstract

We present here a method for the retrieval of the **internal density distribution** of a small body, via a **least-squares inversion** of its gravity field.

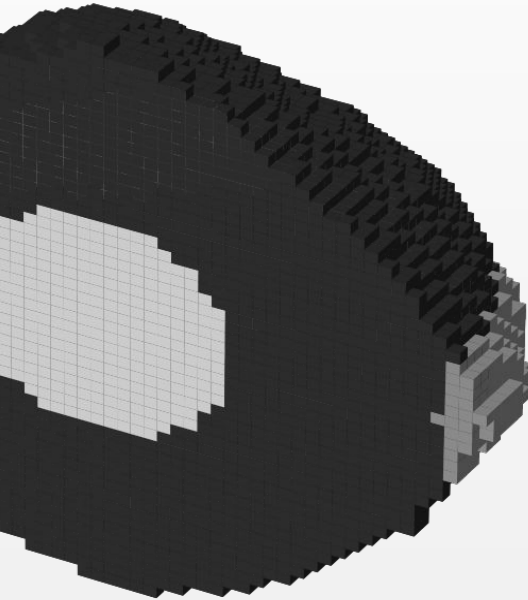
Multiple solutions are **averaged** in order to limit the effect of the a priori density distribution. **Simulations** show successful density map estimations even with little to no initial information on the interior structure of the body.

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Gravity inversion



**Interior
structure**

[1.2]



Gravitational
potential



Spherical harmonics
expansion

**Stokes
coefficients**

C_{nm}, S_{nm}

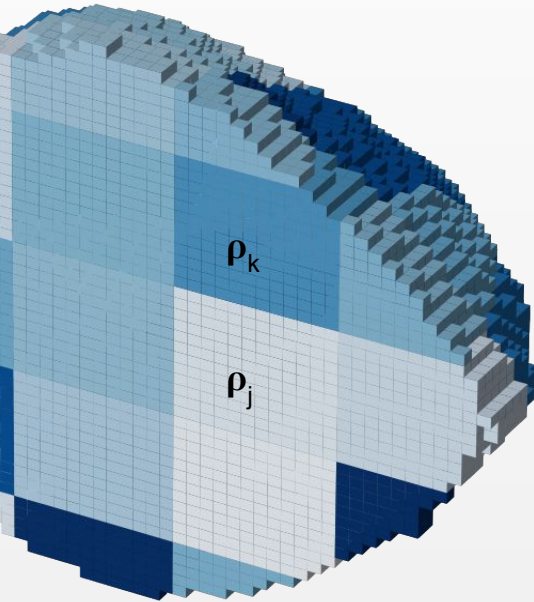
Measured e.g.
via spacecraft
radio-tracking

The body is **discretized** as a collection of cubes

[2]

Gravitational potential as sum of
contributions from each cube

Methodology



Density (ρ) estimated for homogeneous **zones**, so as to have an overdetermined system [\[3,4\]](#):

$$\begin{bmatrix} C_{nm} \\ S_{nm} \end{bmatrix} \xrightarrow{\text{Least-squares inversion}} [\rho_i]_{i \in \text{zones}}$$

Solution depends on the initial zones subdivision (**density map**). Hence:

1. Solution from different maps are **averaged** together [\[5\]](#)
2. An edge-detection algorithm (**Laplacian of Gaussian**) is employed to generate a new map from the averaged solution
3. The **inversion** is performed on the new density map

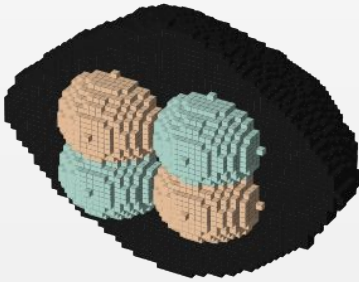
Retrieval simulation

Least-squares inversion

With a range of initial density maps

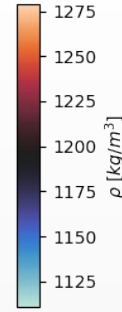
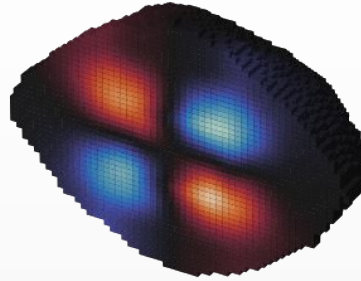
Reference density

Used to simulate a gravity field



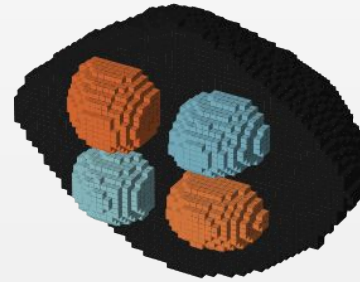
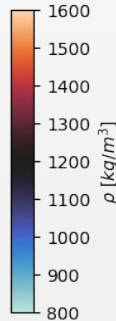
Anomaly detection

By averaging solutions for different initial maps



Inaccurate density values in the averaged solution

Laplacian of Gaussian, providing shape and location of anomalies



Estimated density

Using the derived map for the inversion

Conclusions

Solution appraisal

- Successful detection of the **density anomalies**
- Small inaccuracies in the anomaly **size and shape determination**, which in turn affect the estimated density value
- **Non-unique solution** [6]: in a real scenario, it has to be confirmed by theoretical models or independent investigations

Future work

- Find a better agreement between the retrieved and the nominal models:
 - Improvement of the shape detection algorithm
 - Larger variability in the initial density maps
- Test on real data

Acknowledgements

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References

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