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Droplet distribution models for visibility calculation

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Abstract

In order to address the reduction of visibility due to fog, investigations are undertaken by reproducing the effect of fog by numerical simulation, namely, with a model of light scattering connected to Mie theory. This model needs to integrate droplet distributions or models of particle size in the software. The assumptions for models of particles made so far are based on the work [1] . All the data we have today thanks to experiments in natural fog, allows us to test the application of these models on new data and suggest improvements in case of divergence. This study uses simulations with Gamma and Log-normal laws. The combination of these new simulations with light scattering theory, are able to update models considered in determining road visibility.

1. Introduction

More efficient predictions of fog occurrence and visibility are required in order to improve both safety and traffic management in critical adverse weather situations. Observation and simulation of the fog characteristics contribute to a better understanding of the phenomena and to adapt technical solutions against visibility reduction. The simulation of visibility reduction by fog condition using light scattering model depends on the size and concentration of droplets (see [6]). Therefore it is necessary to include in the software some functions for the droplet distribution model rather than some data file of single measurement. The aim of the present work is to revisit some droplet distribution models of fog ([1]) in order to actualise them by using recent experimental measures. Indeed the models mentioned above were established thanks to experimental data obtained with sensors of 70's. Actual sensors are able to take into account droplets with radius 0.2 µm which was not the case with older sensors. A surface observation campaign was carried out at Palaiseau, France, between 2006 and 2007. These experiments allowed to collect microphysical data of fog and particularly droplet distributions of the fog, thanks to a "Palas" optical granulometer. Based on these data an analysis is carried out in order to provide a droplet distribution model. The first approach consists in testing the four Gamma laws proposed by [1]. The adjustment of coefficients allows changing the characteristics from advection to radiation fog. These functions did not fit the new set of data collected with the Palas sensor. New algorithms based on Gamma and Lognormal laws are proposed and discussed in comparison to the previous models. For a road application, the coefficients of the proposed models are evaluated for a visibility around 50 meters.

Determining the distribution of droplets is the first step to simulate fog and forecast visibility, as all of the fog characteristics come directly from its microphysical structure. Droplet concentration and droplet size are the two critical parameters to be determined. This distribution then allows the calculation of various parameters that will be used to determine the visibility. Once a general shape of fog distributions has been determined, it is possible to propose mathematical models that will simulate the fog distribution for various size and concentrations of droplets. Then by using the Mie theory described in the next section, the corresponding reduction of visibility can be determined.

2. Mie theory

The series of formulas used to calculate the visibility are known as Mie theory (see [5]). The amount of light that won't reach the eye of the observer is related to the extinction coefficient Ke. This coefficient is related the radius r of the droplet through the following formula :

$$K_e = \pi \int_r Q_{ext}(r) r^2 n(r) dr, \qquad (1)$$

$n(r)$ being the number of particles in a given volume. The coefficient Q_{ext} is dimensonless and depends on the droplet size and of the wavelength and varies between 1 and 4 and stabilizes around 2 for drops with a radius of a few microns (see Figure 1).

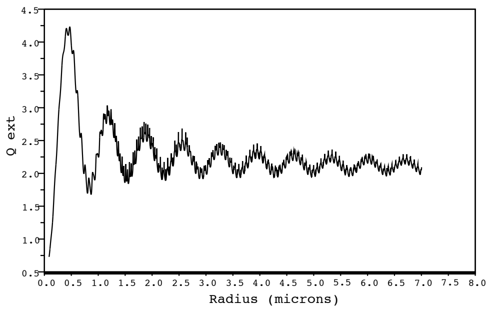

Figure 1: The function Q_{ext} versus the droplet radius for a wavelength equal to 550 nm.

If a granulometer is used to count the number of particles, the formula can be modified as follows:

$$K_e = \pi \sum_{i=1}^{N} Q_{ext}(r_i) r_i^2 N_i \quad (2)$$

where N is the number of classes of radius, N_i the number of droplets in the class of the radius r_i. It should be noted that the extinction coefficient depends on r^2, meaning that even a few large drops will have a considerable influence on fog density. Once the coefficient of extinction has been calculated, the atmospheric visibility V in meters is given by the equation:

$$V = \frac{3}{K_e}. \quad (3)$$

3. Theoretical models of fog distribution

3.1. The considered laws

We propose to revisit, using measurements by current sensors, fog distribution models still used today. This study is based on the work [2] and [1] in which two theoretical models are developped. We propose to confront experimental data collected during the 2006-2007 winter season (Paris, France). This work [2, 1] mentioned laws Log-normal and Gamma amended to model the size of the fog. Denoting $n(r)$ the number of droplets of radius r present in a sample of volume 1 cm^3, the Log-normal law is defined by:

$$n(r) = \frac{N}{r\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln r - \ln \tilde{r})^2}{2\sigma^2}\right), \quad (4)$$

where σ is the standard deviation of the experimental distribution $\ln r$ and $\tilde{r}$ is such that $\ln \tilde{r}$ is the mean of this distribution (see [2]).

The modified Gamma law (see [1]) is expressed as:

$$n(r) = a r^{\alpha} e^{-b r^{\gamma}}, \ r \geq 0, \quad (5)$$

where a, b, α and γ are parameters to be determined.

We also propose in this study to consider a shifted Gamma law: aerosols whose radius is below a certain threshold r_{min} are either inaccessible by experimental measurements or are not considered as fog particles. We then assume that $n(r)$ vanish under a radius r_{min}.

3.2. Gamma laws of Shettle and Fenn

We present in Table 1 the values of the parameters a, b, α and γ of (5) given in [1]. In Figure 2, we present experimental distributions with the 4 models of [1]. We can see an inadequacy between experimental and such theoretical models.

Table 1: Values of the parameters for the four modified Gamma laws of [1].

Fog type	Model	a	α	b	γ
Advection	1	0.06592	3	0.3	1
Advection	2	0.027	3	0.375	1
Radiation	3	2.37305	6	1.5	1
Radiation	4	607.5	6	3.0	1

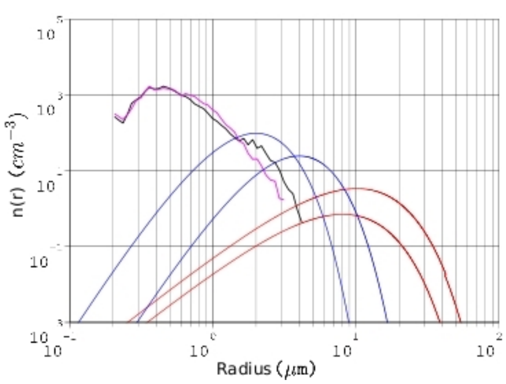

Figure 2: Shettle and Fenn models and experimental distributions.

4. Estimation of the parameters

We describe here the method allowing to estimate the parametrs of (5) from an experimental fog distribution. Let us assume that we know for a fog sampling :

- the total number N of droplets;
- the modal radius r_m;
- the number of droplets N_m for the modal radius.

The modal radius r_m corresponds to the radius for which the derivative of $n(r)$ vanishes. This leads to:

$$a = N_m r_m^{-\alpha} e^{\alpha/\gamma}, \quad (6)$$

$$b = \frac{\alpha}{\gamma} r_m^{-\gamma}. \quad (7)$$

and

$$\frac{1}{\gamma} e^{\alpha/\gamma} \left(\frac{\gamma}{\alpha}\right)^{\frac{\alpha+1}{\gamma}} \Gamma\left(\frac{\alpha+1}{\gamma}\right) = \frac{N}{r_m N_m}, \quad (8)$$

where $\Gamma(x)$, for $x > 0$ is given by:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt. \quad (9)$$

The constraint (8) gives then *a priori* several couples (α, γ), each couple fixing, thanks to (6) and (7), the values of a and b. Assuming $\gamma = 1$ as it is made in [2, 1], only one value for α is possible and by then for a and b.
For a shifted law, that is $n(r) = 0$ if $r \leq r_{min}$ and

$$n(r) = a\left(r - r_{min}\right)^{\alpha} e^{-b(r-r_{min})^{\gamma}}, r \geq r_{min}, \quad (10)$$

the previous calculations can be performed with

$$\frac{1}{\gamma} e^{\alpha/\gamma} \left(\frac{\gamma}{\alpha}\right)^{\frac{\alpha+1}{\gamma}} \Gamma\left(\frac{\alpha+1}{\gamma}\right) = \frac{N}{\left(r_m - r_{min}\right) N_m}. \quad (11)$$

instead of (8).

5. Values of the parameters estimated from experimental data

We are interested in a fog event appeared in the night of 13 to 14 March 2007 on the site of Palaiseau (ParisFog campaign [3, 4]). In this study, we implemented the procedure described in the previous section for a visibility of 50 meters and we give in Table 2 the associated average values coefficients of the shifted Gamma and the Log-Normal laws. For this, 10 samples of experimental distributions have been used.

In Figure 3 we present the 10 experimental distributions and the theoretical ones. We can observed a good fitting of the shifted Gamma law. The calculation of the visibility for the theoretical laws is about 75 m wich is in the same order of magnitude with the target value 50 m. A better theoretical model could be carried out by adding a constraint on the visibility. Indeed, instead of taking $\gamma = 1$ for the shifted Gamma law it could be possible to research new values a, b, α and γ in order to solve (6)-(11) and a new constraint on the visibility given by (1) and (3). This approach will be the topic of futur works.

Table 2: Values of the parameters of the Log-Normal and shifted Gamma laws.

		Log-Norm.		Shifted Gamma		
Vis.	N	$-\ln\tilde{r}$	σ	α	$10^{-3}a$	b
52	1330	0.44	0.59	0.87	30,9	5.06

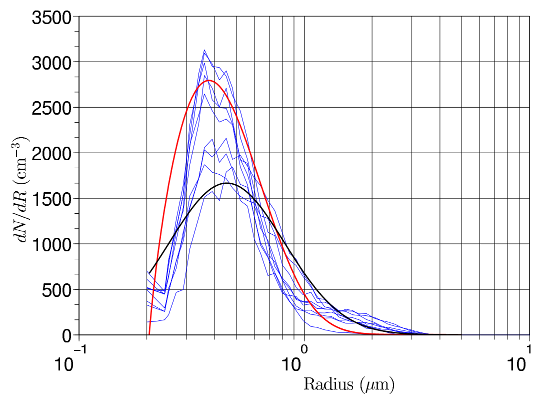

Figure 3: Experimental and theoretical distributions for a visibility around 50 meters.

6. Summary and Conclusions

This paper presents some results of simulations of droplet distribution with Gamma and Log-normal laws. This work uses a set of natural droplet distribution measured during the ParisFog experiment. These data are mostly representative of radiation fog. To extend this simulation, it will be useful to test the model on new set of data representative of advection fog and also of bi-modal distribution. Therefore some new approach with Bayesian statistical theory will be considered.

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