



## **On the Gaussian Mixture Importance Sampling estimator for Bayesian model selection**

Elena Volpi (1), Gerrit Schoups (2), Giovanni Firmani (1), Jasper A. Vrugt (3,4)

(1) University Roma Tre, Department of Engineering, Rome, Italy, (2) Department of Water Management, Delft University of Technology, the Netherlands, (3) Department of Civil and Environmental Engineering, University of California, Irvine, USA, (4) Department of Earth System Science, University of California, Irvine, CA, USA

Bayesian model evidence provides an effective tool for model selection under multiple competing hypotheses. The evidence, or marginal likelihood, measures the probability of the observed data given the model under consideration and it is nothing else than the normalizing constant in the denominator of Bayes theorem.

While it is fundamental for model selection, the evidence is not required for Bayesian inference. Moreover, since it is not particularly easy to estimate in practice, Bayesian model selection via the marginal likelihood has not yet found mainstream use with respect to the more popular information criteria, such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). In fact, the evidence is computed for each hypothesis (model) by averaging the likelihood function over the prior parameter distribution, rather than maximizing it as by information criteria.

The larger a model evidence the more support the model receives among a collection of hypotheses as the simulated values assign relatively high probability density to the observed data. Hence, the evidence naturally acts as an Ockham's razor, preferring simpler and more constrained models against the selection of over-fitted ones by information criteria that incorporate only the likelihood maximum.

To overcome the computational problem, a new estimator of the Bayesian model evidence – coined Gaussian Mixture Importance Sampling (GMIS) – is proposed. GMIS uses multidimensional numerical integration of the posterior parameter distribution via bridge sampling (a generalization of importance sampling) of a mixture distribution fitted to samples of the posterior distribution derived from the DREAM algorithm (Vrugt et al., 2008; 2009). The method provides robust and unbiased estimates of the marginal likelihood without incurring in much larger computational efforts with respect to other commonly used approaches in the literature; some illustrative examples are presented and discussed to demonstrate the robustness and superiority of the GMIS.