

An Iterative method for Beam Hardening Corrections using the Alvarez-Makovski Model

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Summary: We present beam hardening correction methods for iterative reconstruction schemes of X-ray CT. These methods incorporate elements of the Alvarez-Makovski model and are both efficient and scales easily to large data sets.

1. Introduction

Beam hardening correction (BHC) is a problem which has accompanied the field of X-ray Computed Tomography (XCT) for four decades [1]. The persistence of this problem arises from the goals of fidelity and efficiency which are often at cross purposes with each other. Previous methods of reconstruction have prioritised one goal, such as Filtered back-projections (FBP) with post-processing corrections which is speedy but does not fully account for the underlying physics and iterative schemes which model the full physics of the projection process, but require in addition detailed material properties of the scanned object that adds either experimental overhead or additional computational complexity. Failing to provide these details accurately would result in nonsense in the reconstruction. Battling BH artefacts therefore boils down to satisfying the criterion which is given priority by the particular application. However we feel that with suitable simplifications and careful assumptions, higher fidelity can be achieved in efficient methods used in production environments. Therefore, taking the well known paper by De Man et al. [2] as our starting point, we introduce two methods that are both applicable to real life data of considerable size and require only the X-ray spectrum to carry out the correction.

2. Background

Our correction method is simply a way of applying our own forward projection function to a iterative method such as Simultaneous Iterative Reconstruction Technique (SIRT) or Expectation Maximisation Transmission Tomography (EMTR). Since the primary cause of BH artefacts is the polychromatic nature of X-rays which have different material attenuations at different energies, and the monochromatic assumption of the reconstruction technique, the starting point of our model is the Alvarez-Makovski (AM) model, which gives the energy dependence of attenuation in equation (1) below [3]:

$$\mu(E) = K_1 \rho Z^{n-1} \cdot \frac{1}{E^m} + K_2 \rho \cdot f_{\text{KN}}(E) \quad (1)$$

Here the unit-length attenuation $\mu(E)$ at energy E is given by both the effects of Compton scattering (with constant K_2) and the photo-electric effect (with constant K_1). This model assumes that there are no absorption edges such as k-edges near the energies of interest, meaning that it is restricted to samples consisting exclusively of elements of lower atomic number. The atomic number is Z and the density is ρ , f_{KN} is the Klein-Nishina cross section, and usually $n = 4, m = 3$. Since Compton scattering and photo-electric effect scale differently with energy, we need to solve for both in addition to the X-ray spectrum to produce the polychromatic projection

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from the monochromatic attenuation. The problem in applying this model is that of the under-determined system: the two unknowns Z and ρ need to be solved for, while only the projected attenuation and the input spectrum is known. To apply the AM model we therefore need to either obtain further knowledge such as the identity of the material under construction, or make additional simplifications to the model since Z and ρ are not related in a one-to-one relationship for materials and it's impossible to deduce one from the other in the general case.

3. Technique and Results

De Man et al. assumed that at a fixed energy one can deduce uniquely the magnitude of both the Compton scattering and photo-electric effect from the total attenuation. This was done by plotting the magnitude of the Compton scattering and Photo-electric effect against total attenuation for several reference materials. The points between the materials are connected by linear interpolation and the magnitude of the two components for an unknown material can then be deduced by knowing its total attenuation and looking up the graph. These two components are projected separately and the backprojection function is calculated in the typical EMTR scheme. This leads to a total of four projections and four backprojections per iteration, which, while yielding adjoint operators for forward and backprojection, seems excessively complex for many applications. We propose two major simplifications which would require neither more material references nor excessive algorithm runtime. The first of which is by setting K_2 to 0, or ignoring the Compton component, then the attenuation for all objects fall on the same energy scaling curve, and only one forward projection is needed. The second entails the assumption that $Z = k\rho$, which is more or less correct for most elements and some compounds. In this case the attenuation is scaled separately for the two components of the AM model, and two projections are required. Both of these methods use the same backprojection function as the uncorrected iterative scheme, thereby retaining the efficiency for production use, at some cost to fidelity. We present and discuss both simulated and experimental results generated by the ANU CTLab with the two simplified methods. Among the results are satisfactory reconstructions of a coin battery, a pewter object (Figure 1 below) and solid titanium where artefacts were visibly reduced. It is noted that iterative BHC on real experimental data-sets of the size used here (up to $3000 \times 3000 \times 20000$ voxels) has not been attempted before.

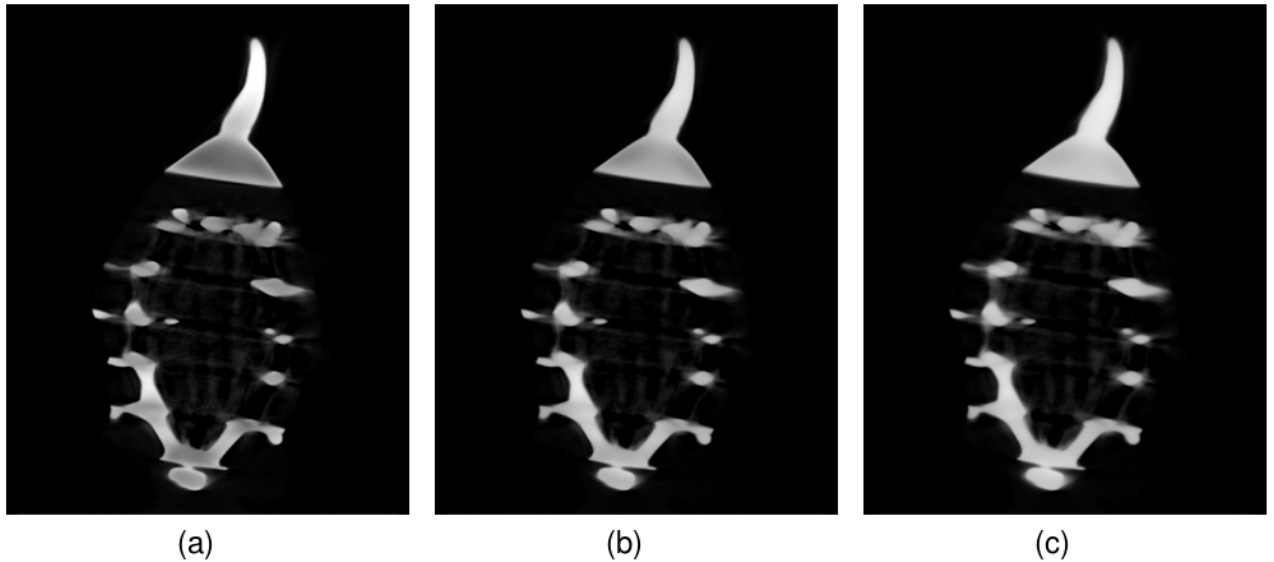


Figure 1: Reconstruction slice of a pewter object, 40 iterations EMTR. (a) Without corrections, (b) With the photo-electric only correction, (c) With the Compton and photo-electric correction

References

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