A LOG-POLAR-BASED METHOD FOR FAST TOMOGRAPHIC RECONSTRUCTION

Viktor Nikitin* [1,2], Fredrik Andersson[2], Marcus Carlsson[2], and Rajmund Mokso[1]

[1]Max IV laboratory, Lund University, Sweden
[2]Center for Mathematical Sciences, Lund University, Sweden

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Summary: We propose a log-polar-based method for tomography data reconstruction with reduced computational complexity of $\mathcal{O}(N^2 \log N)$. A software package to process real measurements by this method has been developed and implemented at the MAX IV cluster, including a simple GUI and is available on github.

INTRODUCTION

The X-ray beam propagation process can be estimated by line integrals over functions representing density or other physical properties of an object along the beam-path. Doing this repeatedly while rotating the sample, we obtain a set of tomographic projections. Projection of the function $f(x)$ for each particular angle θ and distance s of the line to the origin is given by the Radon transform

$$\mathcal{R}f(\theta, s) = \int_{s=x\cdot\theta} f(x)dx = \int_{-\infty}^{\infty} f(s\theta + t\theta^{\perp})dt. \tag{1}$$

Image reconstruction, in turn, refers to the process of producing an object structure from estimates of its line integrals along a finite number of lines. One of the most popular methods for data reconstruction is by means of the filtered back-projection (FBP) method. The method is based on the inversion formula $f = \mathcal{R}^{\#}\mathcal{W}\mathcal{R}f$, where $\mathcal{W}$ is a convolution operator, and where $\mathcal{R}^{\#}$ denotes the back-projection,

$$\mathcal{R}^{\#}g(x) = \int_{S^1} g(\theta, x\cdot\theta). \tag{2}$$

In various situations, it is preferable to use an iterative method for doing reconstruction from tomographic data. A direct implementation of the Radon transform (1) and the back-projection operator (2) has a time complexity of N^3, if we assume that reconstructions are made on an $N \times N$ lattice and that the numbers of samples in s and θ are both $\mathcal{O}(N)$. Iterative reconstruction methods rely on applying the forward and back-projection operators several times, therefore, the improvement of speed in computing the Radon transform and the associated back-projection is important.

A fast log-polar-based algorithm for evaluation of the Radon transform and the back-projection operator was proposed by Andersson et al. [1]. The algorithm reduces computational complexity to $\mathcal{O}(N^2 \log N)$ and shows reasonable accuracy and improved computational costs. The most popular analogues are based on using the Fourier slice theorem, where an interpolation-like procedure has to be performed in the frequency domain, and since data in the frequency domain is rather oscillatory it is required to use high interpolation order. The proposed method also requires interpolation, but this interpolation is performed in the spatial or Radon domains where data are less oscillatory and where it is enough to use low-order interpolations.

METHOD AND IMPLEMENTATION

The idea of the log-polar-based method is to make the change of variables $x_1 = e^{\rho}\cos(\theta)$, $x_2 = e^{\rho}\sin(\theta)$, $s = e^{\rho}$. Then the Radon transform can be evaluated by computing convolutions of the form

$$\mathcal{R}f(\theta, e^{\rho}) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} f(\theta', \rho')e^{\rho'}\zeta\left(\theta-\theta', \rho-\rho'\right) d\rho' d\theta'. \tag{3}$$

*e-mail: viktor.nikitin@maxiv.lu.se

By discretization, periodic extension and suitable geometric transformations, the above convolutions can be approximated by using FFT; see [1] for details. The transfer function ζ can be accurately precomputed. Sampling conditions, so as interpolation schemes to switch between different coordinates (Cartesian (x_1, x_2), polar (θ, s), and log-polar (θ, ρ)) were studied in order to obtain accurate approximation.

The algorithms for the log-polar-based reconstruction were implemented on both CPU and GPU platforms. In particular, the GPU platform can favorable be used for the proposed method due to the fact that linear interpolation is hard-wired on GPUs, meaning that it has the same computational cost as direct memory access. Moreover, cubic order interpolation schemes can be constructed by combining linear interpolation steps and this provides important computation speedup. The second reason of choosing the GPU platform is the existence of high-performance libraries to perform FFT and matrix-vector computations. Finally, GPUs are well-adapted for iterative schemes, where during all iterations the processing data can be stored in device memory.

The proposed software package works via a C++ extension interface for Python, where the C++ code is accelerated by using NVIDIA CUDA technology. A proposed simple GUI interface works with HDF5 files and automatically performs flat and dark field corrections. Functionality includes choosing of the inversion filter type, extra padding in radial direction, and indexes of slices to reconstruct.

RESULTS

In Table 1 we summarize performance and accuracy results for the reconstruction by the filtered back-projection using the log-polar-based method (LP), Astra tomography toolbox [4], NFFT3 library [3], and IRT toolbox [2]. Accuracy levels were calculated by taking normalized difference between the exact expression for the filtered Shepp-Logan phantom and the result after applying the back-projection to the exact filtered Radon data. The Radon data has sizes $(N_\theta \times N_s) = (\frac{3}{2}N \times N)$, where $(N \times N)$ is the size of the reconstructed image.

Table 1: Accuracy and computational times of the back-projection

Package	Method	Device	Accuracy	Time scalability		
			$N = 512$	$N = 512$	$N = 1024$	$N = 2048$
LP	log-polar	GPU	0.031	2.9e-03s	1.2e-02s	4.5e-02s
ASTRA	FBP	GPU	0.047	7.1e-03s	4.3e-02s	2.8e-01s
LP	log-polar	CPU	0.031	2.6e-02s	1.1e-01s	4.0e-01s
NFFT3	Fourier	CPU	0.022	1.7e-01s	6.9e-01s	3.2e+00s
NFFT3	Fourier(pseudo-polar)	CPU	0.020	2.2e-01s	9.2e-01s	4.6e+00s
IRT	Fourier	CPU	0.021	3.7e-01s	1.5e+00s	6.3e+00s

According to Table 1 the proposed program on GPU is about 5 times faster than ASTRA, and about 2 or more magnitudes faster than the other CPU based methods. The program also demonstrates a reasonable accuracy level for reconstructions. The accuracy of the Fourier-based methods controlled at user level, but where increased accuracy leads to increased computational cost. For the same accuracy level the log-polar-based method has a higher gain in performance compared to the Fourier-based methods.

The package was successfully tested at MAX IV HPC cluster on the node with GPU Tesla K80 to process real tomography data from the Swiss Light Source. The developed software package are publicly available: https://github.com/math-vrn/guirec. In further work, we plan to extend the package functionality by iterative methods using the proposed projectors and a method to reconstruct time resolved tomographic series.

References

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