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# A new covariant form of the equations of geophysical fluid dynamics and their structure-preserving discretization

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For reliable climate simulations performed with Earth system models, an accurate simulation of the flow features of atmosphere and ocean is crucial. Beside plenty of other processes, these flows governed by the laws of geophysical fluid dynamics (GFD) are usually determined with computational models. The quality of the latter depend on the physical models that describe the phenomena to be simulated, i.e. on the sets of analytical equations, and on how these analytical equations are approximated (or discretized) in order to make them suitable for computational calculations. Such discrete models should inherit certain conservation properties from the analytical equations, such as mass and energy conservation and the preservation of certain flow features. Often already the formulation of the analytical equations provides a guideline to find the corresponding approximated equations.

I introduce a new formulation of the analytical equations of GFD that allows to find structure-preserving approximated equations for computational calculations. The resulting discrete model is structure-preserving in the sense that the calculated approximated solutions are mass and energy conserving and that the discrete velocity fields consist of divergence-free and rotation-free parts, analogously to the analytical fields. The latter property is important to preserve certain flow features also in the discrete case and is often refer to as Helmholtz decomposition.

The new formulation consists in a separation of the momentum and continuity equations into a metric-free (topological) and a metric-dependent part. In the metric-free part, only neighborhood relationships are taken into account, which allow, for instance, to indicate the boundary of an area or to find its neighboring regions. The information about the metric structure, such as length, area, angle, etc., are confined to the metric-dependent equations. This splitting of the equations has been enabled by introducing additional prognostic variables and presenting all variables with differential forms instead of vector fields. The latter fact makes these equations covariant, i.e. their form is invariant under coordinate transformations. This new structure reveals important geometrical features of the equations of GFD, for instance, velocity and vorticity are more adequately described by differential forms than vector fields. A better understanding of the equations, in turn, allows more accurate approximations.

On the basis of this split formulation, I develop a new systematic discretization methodology, using tool of discrete differential geometry, that leads to approximated model equations that automatically preserve important conservation properties. I illustrate this methodology on the example of the linear shallow-water equations, for which I derive a finite difference approximation that conserves mass and energy, while preserving the Helmholtz decomposition, on arbitrarily structured C-grids. Using my structure-preserving approach thus minimizes the uncertainties in the simulation of geophysical scenarios, because approximated models that do not preserve such conservation properties usually require additional stabilizations, which may badly impact on the model accuracy.