ICYESS2013-61-1
Interdisciplinary Conference of Young Earth System Scientists
Hamburg, Germany, 22 – 25 September 2013

Goal-orientated Error Estimators for Stochastic Finite Elements

J. Kraemer
Fakultät für Mathematik, Karlsruher Institut für Technologie, Karlsruhe, Germany (juliankr@gmx.net)

In my diploma thesis I develop adaptive numerical methods to quantify uncertainties for problems modelled by partial differential equations. The latter are used to describe many physical phenomena in nature, such as for example flows in the ocean or in the atmosphere. A huge problem, however, is the uncertainty introduced by model and parameter errors due to for example measurement errors or a lack of available data. In the past, this uncertain influences were ignored, but as the precision did increase, it became inevitable to model the errors too.

Typically, even in the purely deterministic case, the discretization of such problems leads to a very large number of equations, which is significantly increased even further if stochastic (uncertain) models are taken into account. This necessitates the development of effective approaches, which still allow for a high accuracy in the numerical computation.

In numerical simulations, the underlying domain is typically divided into cells and the solution of the system is only evaluated at certain points of the cells, since one can not evaluate all infinitely many points in a continuous domain. These cells may have different sizes. One can show that the division into these cells leads to a converging solution if the cells widths decrease. However, this leads to a growing number of equations which have to be solved. Therefore one has to find a way to partition the domain optimally into cells, which is intended by adaptive refinement strategies.

Adaptive strategies follow a simple procedure. First one takes a grid which consists of equally sized cells and computes a solution on it. After that the error is estimated on each cell individually and the cells with the highest estimated error are selected and refined, which means that each selected cell is divided into a number of smaller ones. This algorithm is performed several times until a stopping criteria is fulfilled. This assures, that the points where the solution is computed are distributed in an optimal manner.

In my case, I investigate the so called goal-oriented adaptivity approach where a certain user-defined physical feature of the solution is of interest. For example, in weather forecasting not the domain and all its properties like wind speed and temperature at all points is of interest, but only one certain feature, like the rainfall, is evaluated. This focusing can be exploited and allows for highly efficient discretizations. The theory for the deterministic case is already well known, but there is little work yet published on the combination of the goal-oriented adaptivity with stochastic uncertainties. In my work I address the question how such goal-orientated techniques can be used to steer the adaptation of the cells in the presence of uncertainties in the data for the spatial and stochastic domain. This could lead to a new approach which could accelerate the simulation of flows and therefore allow a higher accuracy while using the same hardware resources.