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Development of a Solution Scheme Applying the Multigrid Method for Three-Dimensional Radiative Transfer Equation

H. Ishida
Graduate School of Science and Engineering, Yamaguchi Univ., Ube, Yamaguchi Pref., Japan (ishidah@yamaguchi-u.ac.jp)

This study proposes a new scheme to explicitly solve the three-dimensional (3D) radiative transfer equation applying the multigrid method. The radiative transfer calculation in the atmosphere involving spatial inhomogeneities, which is always caused by cloud existence, is necessary in order to improve satellite remote sensing and estimation of radiative energy budget in atmospheric models. However, the 3D radiative transfer often needs a large resource for computation compared to a plane-parallel approximation. Therefore, development of an efficient scheme for 3D radiative transfer caluclation is required.
It is necessary for an explicit solution of radiative transfer to discretize the equation for separation of the independent variables. We employ the spherical harmonic expansion to discretize the terms that depend on the direction of radiance (e.g., angles). This discretization scheme provides a facile means to obtain radiances in an arbitrary direction. For the discretization of the terms depending on the spatial coordinates, we apply grid partitioning and the finite-volume method for the partial differential terms. Because the 'velocity' of a radiance cannot be determined uniquely, the upwind difference scheme is modified to make the radiative transfer equation determinate. Finally, the radiative transfer equation results in a vector-matrix form as a simultaneous linear equation, which is able to be solved with an iteration scheme such as the successive over-relaxation method.
The vector-matrix equation always includes large vectors and matrices, according to the number of discretization terms. One of the effective iteration schemes for such a equation is the multigrid method. In general, iterations on a fine grid quickly reduces high frequencies in smooth errors, whereas low frequencies are reduced very slowly. If the iteration is carried out on a coarser grid, low frequencies can be reduced efficiently, resulting in convergent solution with fewer iterations. The multigrid method involves two operations; 'restriction', which means mapping from the original (fine) grid to a coarser grid, and 'prolongation', which means interpolation from a coarse grid to a finer grid. However, the general multigrid method cannot be applied to the modified upwind difference scheme. Therefore, we investigated realizability conditions for the multigrid method and developed a restriction and prolongation algorithm to adapt the radiative transfer.
We will explain the algorithm of the solution applying the multigrid method. We will also present some examples of radiative transfer calculation and compare other methods for verification, suggesting our scheme is efficient and suitable for estimation and understanding of radiation fields with clouds.