



Solution anisotropic part elimination of vectorial radiative transfer equation for arbitrary medium geometry

V. Budak and O. Shagalov

Radio Engineering and Electronics Institute, National research university Moscow Power Engineering Institute, Moscow, Russian Federation (BudakVP@mpei.ru)

The discretization of the vectorial radiative transfer equation (VRTE) based on replacing the scattering integral by a finite sum is required for its numerical solution. The physical basis of transport theory is the ray approximation, which inevitably generates spatial and angular singularities of solution and makes it impossible to replace the integral by the sum. Therefore, the VRTE solution is necessarily represented as the sum of the anisotropic part, including all the singularities of the exact solution and determined analytically, and the smooth part, determined numerically from the boundary value problem for discrete VRTE. Discretization leads VRTE to a system of differential equations. In case of a turbid medium slab in accordance with the form of the VRTE differential operator this system is linear of the first order with constant coefficients that allows an analytic solution in the matrix form for the solution smooth part.

All modern algorithms for solving VRTE for the medium slab are based on such a method. The differences among these algorithms are determined by the method of the solution anisotropic part elimination. The most effective method of determining the anisotropic part is a small-angle modification of the spherical harmonics method (MSH), based on the analysis of the solution angular spectrum: near the angular singularity the radiance spectrum (the representation as the series of spherical harmonics) is a slow monotonic function of the harmonic index. MSH determines with high accuracy the solution in the forward hemisphere of directions and uncertainly in the backward hemisphere.

The smooth part, in fact, defines MSH in the backward hemisphere. This method loses its effectiveness in the presence of peculiarities of the phase function for wide scattering angles – the fine structure of scattering. The best method for accelerating the convergence to improve fine structure is the method of iterations. The first iteration already significantly improves the solution, and for the same accuracy it is required to use, for example, in the method of discrete ordinates a much smaller number of streams.

The fulfilled analysis allows formulating a method for solving VRTE for nonplanar geometry: broken clouds, twilight sensing, and lidar. At the first stage, the VRTE solution is found by MSH, and then the source functions for the smooth part are determined. Searching the solution smooth part is carried out by the iteration method, which allows redefining the MSH for the backward hemisphere with a minimum number of streams, such as two-stream approximation. The carried out investigations show that for the slab the error of two-stream approximation for the solution smooth part with the first iteration does not exceed 1%. The study of boundary value problem of broken clouds by way of the example of the slab with a cylindrical hole is carried out. It is not required more than 8 iterations in case, when the hole diameter not exceeding its height. The standard approach the quasi-spherical atmosphere based on the independent pixel approximation cannot be applied to solving the problems of twilight. The proposed method is devoid of these defects.