



Investigation of water wave breaking phenomena: experiment and theory

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The phenomenon of water wave breaking and the link between the wave breaking and the velocities of water particles on the free surface are investigated. Results of experiments carried out in a laboratory flume are presented, and then compared with predictions of a theoretical model that has also been outlined in the paper. The experiments have been conducted in a 64 m long water channel at the Institute of Hydro-Engineering of the Polish Academy of Sciences in Gdansk, Poland. The experiments have been focused on measurements of Lagrangian velocities of fluid particles on the free surface of water. The motion of the fluid was induced by a piston-type wave maker that generated short trains of mono- and bi-chromatic waves propagating in water of a mean-level depth of 40 cm, in which an underwater inclined ramp, of a slope of 10 per cent and a height of 30 cm, was mounted. The height of a generated wave was adjusted in such a way that the wave breaking occurred in a chosen location over the ramp. The wave breaking was of a spilling type. The fluid particle velocities were measured by putting floating markers (small plastic beads of densities very close to that of water) on the free surface of water, and then by recording their movements by means of a camera during the process of wave breaking. The displacements of the markers in time were determined from the analysis of their positions in successive frames of the film.

In the paper also a theoretical model describing the propagation of waves over an uneven bottom is presented. The model is formulated in the Lagrangian variables. A key simplification on which the proposed theory is based is that the vertical displacements of fluid particles are related to an assumed variation of the horizontal displacements, the continuity equation, and the boundary condition at the bottom. The momentum equation, derived by applying a variational principle and making use of the latter assumptions, is equivalent to the Boussinesq approximation of fourth order in kh (where k is the wave number and h is water depth). A direct method of variational calculus was followed, in which the action integral is minimized. A dispersion relation resulting from the equation of motion, as well as so-called shoaling coefficients, were compared with theoretical results known from the literature on the subject. An assumed form of the horizontal displacement distribution was expressed in terms of three parameters. These parameters were selected in such a way that the obtained dispersion relation, the shoaling coefficient, and - in the case of a constant water depth - also the amplitudes of the second and third harmonics, become close to the results obtained from the Stokes theory. The proposed model has proved to be capable of a better description of non-linear wave effects than the corresponding approximation of the same order derived by using the Eulerian description.

Based on the theoretical formulation, a finite-element model has been developed for numerical simulations of the wave phenomena considered. The results of calculations have been compared with experimental data, showing a very good agreement for long waves. For shorter waves, $L/h < 4$ (L is the wave length), some discrepancies occur, which are due to the fact that a non-linear correction term in the dispersion relation is insufficiently large, so that for large-amplitude waves their experimentally measured speeds are greater than those calculated. The proposed model has also been used to determine the velocities of water particles on the free surface, and the comparisons between the experimental and numerical results are presented in the paper. In addition, also two basic wave-breaking criteria, namely the dynamical and the kinematical Stokes stability condition, have been investigated. The analysis has shown that crucial is the dynamical condition, since the kinematical condition has never been reached in our investigations.