



Experimental and theoretical analysis of long waves transformation on a sloping beach

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Transformation of long water waves on a sloping beach has been investigated, both experimentally and theoretically. Experiments have been conducted in a 64 m long and 0.6 m wide laboratory flume at the Institute of Hydro-Engineering, Polish Academy of Sciences, in Gdansk, Poland. Plane monochromatic waves have been generated by a piston-type wave maker situated at one end of the flume, and the sloping beach has been modelled by an inclined rigid ramp, of the slope equal to either 10 or 15 per cent, placed at a distance of 12 m from the generator wall. The water wave parameters have been recorded by a set of gauges installed along the flume, both in its constant- and varying-depth parts. Additionally, the run-up of the wave has been measured by a special conductivity gauge mounted on the ramp along the wave propagation direction. The experiments have been carried out for a wide range of wave lengths and amplitudes, falling, however, into the long-wave regime.

The theoretical analysis of the wave propagation phenomenon has been performed by solving the problem in Lagrangian coordinates, since this permits simple formulation of boundary conditions on the moving boundaries of the fluid domain. However, the price for it is a more complicated structure of equations describing the fluid motion, compared to more traditional approaches based on the Eulerian description. In order to simplify the analysis, the shallow water approximation is applied. An essential simplification, on which the theoretical formulation proposed in this work is based, is a kinematical assumption that fluid motion is 'columnar'; that is, the vertical material lines of fluid particles remain vertical during the motion. Fundamental equations of the theoretical description of the problem have been derived by following the Hamilton principle. Owing to the above kinematical assumption on the fluid motion, all the integrands in the action integral are expressed in terms of only the fluid horizontal displacements and time, together with relevant partial derivatives. The variation of the action integral provides the momentum equation and natural boundary conditions. In order to solve the resulting non-linear partial differential equation, the spatial derivatives are approximated by finite differences, which yields a system of ordinary differential equations with respect to time. The latter equations are integrated numerically by applying a fourth-order Runge-Kutta method.

A comparison of the theoretical results with data obtained in experiments shows that the proposed formulation gives solutions of satisfactory accuracy in almost entire fluid domain, except for a small vicinity of a shore line. The proposed theoretical method also enables good estimation of the fluid free surface elevation for steep waves propagating in domains with a small continuous variation of depth.