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Analytic estimates of tsunami amplitude near the beach

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We show analytic formulas which give explicitly the amplitude of a tsunami wave near the beach under the hypothesis that the depth profile is proportional to the distance from the beach. These formulas are derived using an asymptotic expansion of the wave equation in the parameter $\mu = \frac{a}{L}$, where a is the width of the initial perturbation and L is the length of the basin. The asymptotic expansion is following the Maslov approach which gives an analytic representation of the solution as an explicit function of the initial perturbation by means of the Maslov canonical operator. We generalize this approach to the case of the singularity given by the beach. The initial perturbation is

$$V(y) = V^0(T(\theta y)) \quad T(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{1}$$

$$V^0(y) = \frac{1}{(1 + \frac{(y_1)^2}{(b_1)^2} + \frac{(y_2)^2}{(b_2)^2})^{\frac{3}{2}}}$$

$$V(y) \to V(\frac{y-a}{\mu})$$

$T(\theta)$ is a rotation of the initial perturbation which is not of a gaussian type, differently from the other cases. The reason of this choice is that the Fourier transform of this perturbation is rather simple. The amplitude is an explicit function of the parameters of the perturbation and so it is possible to find the initial shape of the wave from the measures of the waves in some points of the ocean and then reconstruct the wave amplitudes in real time.

$$u = \mu \mathrm{Re} \frac{\tau^0(\mathrm{x_2}, \mathrm{t}) - \mathrm{t} + \mathrm{i}\mu\beta(\psi^0(\mathrm{x_2}, \mathrm{t}))}{(-4\mathrm{x_1} + (\tau^0(\mathrm{x_2}, \mathrm{t}) - \mathrm{t} + \mathrm{i}\mu\beta(\psi^0(\mathrm{x_2}, \mathrm{t})))^2)^{3/2}}. \tag{2}$$

The function β contains the information of the initial amplitude

$$\beta(\psi) = \sqrt{b_1^2 \cos^2(\psi) + b_2^2 \sin^2(\psi)}$$

τ^0, ψ^0 are parameters used for defining the front in the position x_1, x_2. τ^0, ψ^0 are the initial conditions of the system of characteristics; the solution of this system gives the front at different times. This approach can be used for a real time tsunami alarm system.

References

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